AIR FORCE INSTITUTE OF TECHNOLOGY

AIR UNIVERSITY
UNITED STATES AIR FORCE

SCHOOL OF ENGINEERING

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

AF-WP-O-MAY 60 3,000

A RELAY DEVICE TO DEMONSTRATE RESIDUE-TO-DECIMAL NUMBER CONVERSION

THESIS

Presented to the Faculty of the School of Engineering

The Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the

Master of Science Degree

in Electrical Engineering

by

Arthur James Altenburg, B. S.

Capt., USAF

GE/EE/62-1

Graduate Electronics

December 1962

Preface

This report presents the development of a solution to the problem of residue-to-decimal number conversion, and the method of application of the solution to a relay converter, to demonstrate its practicability. The converter has been retained by the laboratory which sponsored this project, the Bionics and Computer Branch, Electronic Technology Laboratory, ASD.

Most of the actual solution to the conversion problem is, to the best of my knowledge, a result of my own efforts; and any errors that may appear are entirely my own. I most gratefully acknowledge my indebtedness, however, to the personnel of the Computer Division of the Radio Corporation of America, Burlington, Massachusetts, who imparted to me the ideas which later led to the algebraic sign determination capability, and to Mr. E. Day of that organization, with whom these ideas originated.

I wish also to acknowledge my indebtedness to Capt. F. M. Brown of the Institute of Technology, and to Capt. A. L. Calton, Mr. T. S. Gerros, and Mrs. L. H. Tackett of the Bionics and Computer Branch. Each contributed, in his own way, invaluable guidance and assistance.

And finally, to my wife, for her continued patience and encouragement, I offer my most heartfelt thanks.

Arthur J. Altenburg

Contents

List of Figures

List of Figures

List of Tables

Abstract

A residue can be described as the units digit of a number when expressed in a polyadic system with a radix equal to the modulus; a residue number system can be described as the residue-coded units digit of a polyadic system with a radix equal to the product of the moduli. The residue-to-decimal conversion problem can be restated in terms of capacity, algebraic sign determination, and machine to human language translation. The capacity factor can be subordinated by a shift of responsibility from the device to its operator. Solutions to the remaining factors, based on the above notions of residue number, require that variations in assigned ranges of positive and hegative numbers be limited to increments of the limiting modulus, the smallest modulus of the system. Problem solution logic, stated in terms of switching functions for the 3-4-5 residue number system, is duplicated by relay circuits to demonstrate application of the principles developed. Residue numbers in which the limiting modulus residue is zero, called base residue numbers, can be identified by the residues of the other moduli for each base residue number, called base residues. Reduction of a residue number to its base residue number by subtraction of the limiting modulus from each of the residues determines the incremental range of the limiting modulus within which the given number lies. Comparison of this range with assigned negative number ranges determines the algebraic sign of the number. Further range

comparisons with the units, tens, and higher-order decimal digits represented by each range, coupled with inter-and intra-range comparisons to resolve ambiguities, provides the required language translation for problem solution. A significant simplification in units digit determinations for residue number systems containing one even modulus, and another modulus some multiple of five through reductions of the residues of the specified moduli to residues Mod 2, and Mod 5, respectively, is achieved.

Report on

A RELAY DEVICE TO DEMONSTRATE RESIDUE-TO-DECIMAL NUMBER CONVERSION

I.. Introduction

Residue number systems possess a strong appeal for general-purpose digital computer applications because, through their use, arithmetic operations can be performed without accomplishment of the carry or borrow functions, resulting in a significant decrease in arithmetic organ time and equipment requirements.

Several problems must be solved, however, before residue number system techniques can be applied to general-purpose computers. One of these problems is the unique conversion of numbers from an apparantly unordered residue number system to the decimal system, for subsequent interpretation and use by a human operator.

The objective of this probject is the development and demonstration of a practical method of residue-to-decimal number conversion, applicable for any range of positive and negative numbers, in any residue number system.

The Chinese Remainder Theorem is a well-known algorithm by which a number can be converted from a residue number system to the decimal system. The algorithm is an extremely unwieldy one, however, which has not been successfully mechanized for general-purpose applications, and which, at any rate,

has a direct conversion capability limited to positive numbers. Reference is made to the theorem as a proof of certain developmental concepts, but because of its complexity and limitations, the algorithm, itself, is abandoned as a basis for solution of the thesis problem. Instead, the conversion problem is reduced to three more fundamental problems, which are attacked utilizing a slightly different approach to the concepts of residue number systems than that conventionally taken.

This report deals with the development of a general solution to the residue-to-decimal number conversion problem, and the construction of a relay device to demonstrate an application of the general solution to a specific residue number system.

The report consists of five sections. Section II contains a brief summary of residue number systems. Section III contains a description and an analysis of the residue-to-decimal number conversion problem. Section IV contains a detailed account of the general solution to the problem, coupled with a description of the circuitry employed to apply the general solution to a specific residue number system. Section V contains a brief discussion of the problems remaining to be considered before residue number system techniques can be applied to general-purpose computers, and conclusions.

II. Residue Number Systems

"Number - A symbol or word, or a group of either of these, showing how many, or what place in a sequence."

Webster's New World Dictionary of the American Language. The World Publishing Company, 1958.

Residues

If x and m are integers with $m \neq 0$, there exist unique integers q and r, with $0 \leq r < |m|$, such that

$$x = qm + r$$

where q, the quotient of x divided by m, is the integral part of x divided by m

$$q = \left[\frac{x}{m}\right]$$

and r, the remainder after division of x by m, is m times the fractional part of x divided by m (Ref 5:95)

$$r = m \left\{\frac{x}{m}\right\}$$

The expression for x can, therefore, be rewritten in the form

$$x = m\left[\frac{x}{m}\right] + m \left\{\frac{x}{m}\right\}$$

Two integers x and y which have the same remainder after division by m

$$\left\{\frac{x}{m}\right\} = \left\{\frac{y}{m}\right\}$$

are said to be congruent modulo m, conventionally written

$$x \equiv y \pmod{m}$$

Obviously, then, x is congruent to r modulo m, so that, with r constrained to be positive, the equations

$$x \equiv r \pmod{m}$$

and

$$0 \leq r < m$$

define the reduction of x to its least positive residue, modulo m, written as (Ref 1:1-1)

$$|x|_m = r$$

The expression for x, then, can again be rewritten in the form (Ref 1:1-2)

$$x = m\left[\frac{x}{m}\right] + |x|_m$$

This expression for x, however, is immediately recognizable as a polyadic representation of x in the form

$$x = a_0 + a_1m + a_2m^2 + a_3m^3 + \ldots + a_nm^n$$

where

$$|x|_m = a_0$$

and

$$\left[\frac{x}{m}\right] = a_1 + a_2m + a_3m^2 + \ldots + a_nm^{n-1}$$

and where a_0 is seen to be the units digit of x when expressed in the m-adic number system. Thus, the residue of an integer modulo m is the units digit of that integer, when expressed in the m-adic system.

For example, Table 1 is a table of decimal integers, 0 through 71, with their triadic (Base 3), tetradic (Base 4), and quintadic (Base 5) equivalents. With the overlay in place, showing only the decimal integers and the units digits of their equivalents, the residue of any decimal integer, 0 through 71, (Mod 3), (Mod 4), or (Mod 5), is immediately apparant. Thus, the residue of 9 (Mod 4) is 1; the residue of 23 (Mod 3) is 2; and the residue of 49 (Mod 5) is 4.

Residue Numbers

A residue number is generated by arranging the residues of a number for any n moduli m_1 , m_2 , ... , m_n in a grouping such that each residue is identified with its particular modulus. Thus, given the residue number (1, 2, 3) in the 3-4-5 residue number system, the residue (Mod 3) of the number is 1; the residue (Mod 4) is 2; and the residue (Mod 5) is 3. Referring to Table 1 with the overlay in place, in the 3-4-5 residue number system, the residue number of 58 is (1, 2, 3), of 47 is (2, 3, 2), and of 13 is (1, 1, 3).

The residue of a number modulo m has been shown to be the units digit of that number in the m-adic number system, so there are m possible residues of a number modulo m. Now, the General Principle of Permutations and Combinations states

> "If a thing may be done in M ways, and (after it has been done in any one of these ways) a second thing may then be done in N ways, then the total number of ways of doing the two things is MN." (Ref 2:190)

Decimal	Base 3	Base 4	Base 5	Decimal	Base 3	Base 4	Base 5
	$3^3 3^2 3^1 3^0$	$4^3 4^2 4^1 4^0$	$5^2 5^1 5^0$		$3^3 3^2 3^1 3^0$	$4^3 4^2 4^1 4^0$	$5^2 5^1 5^0$
0	0	0	0	36	1 1 0 0	2 1 0	1 2 1
1	1	1	1	37	1 1 0 1	2 1 1	1 2 2
2	2	2	2	38	1 1 0 2	2 1 2	1 2 3
3	1 0	3	3	39	1 1 1 0	2 1 3	1 2 4
4	1 1	1 0	4	40	1 1 1 1	2 2 0	1 3 0
5	1 2	1 1	1 0	41	1 1 1 2	2 2 1	1 3 1
6	2 0	1 2	1 1	42	1 1 2 0	2 2 2	1 3 2
7	2 1	1 3	1 2	43	1 1 2 1	2 2 3	1 3 3
8	2 2	2 0	1 3	44	1 2 2 2	2 3 0	1 3 4
9	1 0 0	2 1	1 4	45	1 2 0 0	2 3 1	1 4 0
10	1 0 1	2 2	2 0	46	1 2 0 1	2 3 2	1 4 1
11	1 0 2	2 3	2 1	47	1 2 0 2	2 3 3	1 4 2
12	1 1 0	3 0	2 2	48	1 2 1 0	3 0 0	1 4 3
13	1 1 1	3 1	2 3	49	1 2 1 1	3 0 1	1 4 4
14	1 1 2	3 2	2 4	50	1 2 1 2	3 0 2	2 0 0
15	1 2 0	3 3	3 0	51	1 2 2 0	3 0 3	2 0 1
16	1 2 1	1 0 0	3 1	52	1 2 2 1	3 1 0	2 0 2
17	1 2 2	1 0 1	3 2	53	1 2 2 2	3 1 1	2 0 3
18	2 0 0	1 0 2	3 3	54	2 0 0 0	3 1 2	2 0 4
19	2 0 1	1 0 3	3 4	55	2 0 0 1	3 1 3	2 1 0
20	2 0 2	1 1 0	4 0	56	2 0 0 2	3 1 0	2 1 1
21	2 1 0	1 1 1	4 1	57	2 0 1 0	3 2 1	2 1 2
22	2 1 1	1 1 2	4 2	58	2 0 1 1	3 2 2	2 1 3
23	2 1 2	1 1 3	4 3	59	2 0 1 2	3 2 3	2 1 4
24	2 2 0	1 2 0	4 4	60	2 0 2 0	3 3 0	2 2 0
25	2 2 1	1 2 1	1 0 0	61	2 0 2 1	3 3 1	2 2 1
26	2 2 2	1 2 2	1 0 1	62	2 0 2 2	3 3 2	2 2 2
27	1 0 0 0	1 2 3	1 0 2	63	2 1 0 0	3 3 3	2 2 3
28	1 0 0 1	1 3 0	1 0 3	64	2 1 0 1	1 0 0 0	2 2 4
29	1 0 0 2	1 3 1	1 0 4	65	2 1 0 2	1 0 0 1	2 3 0
30	1 0 1 0	1 3 2	1 1 0	66	2 1 1 0	1 0 0 2	2 3 1
31	1 0 1 1	1 3 3	1 1 1	67	2 1 1 1	1 0 0 3	2 3 2
32	1 0 1 2	2 0 0	1 1 2	68	2 1 1 2	1 0 1 0	2 3 3
33	1 0 2 0	2 0 1	1 1 3	69	2 1 2 0	1 0 1 1	2 3 4
34	1 0 2 1	2 0 2	1 1 4	70	2 1 2 1	1 0 1 2	2 4 0
35	1 0 2 2	2 0 3	1 2 0	71	2 1 2 2	1 0 1 3	2 4 1

Table 1

Triadic, Tetradic, and Quintadic Equivalents of Decimal Integers 1 through 71.

Therefore, if the moduli m_1 , m_2 , ... , m_n are pairwise prime (i.e., no two moduli have a common divisor other than 1, so that their least common multiple is $[m_1,m_2] = m_1m_2$) the number of distinct residue groupings is

$$M = \prod_{i=1}^{n} m_i$$

Then, if the moduli m_1 , m_2 , ... , m_n are pairwise prime, with the residue of zero for any modulus defined as zero, there is a unique residue number representation for every number in the range 0 to (M - 1). Furthermore, the Chinese Remainder Theorem (Ref 4:246) states that every decimal number in the range 0 to (M - 1) will be uniquely determined by one of the residue groupings.

Thus, for the (pairwise prime) moduli 3-4-5, there are M = 60 unique residue number representations. Referring to Table 1 with the overlay in place, there is a distinct residue representation for each number 0 to 59, with the sequence of residue numbers apparantly repeating itself in the next range of M = 60 numbers. An extension of the table would illustrate that the sequence of residue numbers is, in fact, repetitive over every range of M = 60 numbers.

A repetitive sequence of digits over a range of M numbers is the characteristic of the units digits of an M-adic number system, however, so utilizing the definition of a number as a symbol or group of symbols showing how many, or

what place in a sequence, a residue number system may be defined as a sequence of numbers, coded with the residues of numbers for any n moduli m_1 , m_2, ... , m_n , which constitutes the units digits of an M-adic number system, where

$$M = \prod_{i=1}^{n} m_i$$

Then, since the units digits of an integral polyadic number possess zero-order positional significance (Ref 3:61), a residue number system is an unordered system only in that it possesses zero-order positional significance. If the residue-coded coefficients of the M's digits of a number could be determined, they would possess first-order positional significance, and they would form, in combination with the residue-coded units digits of the original residue number system, a fully ordered system.

The "reduction" of a number from any polyadic number system to a residue number system, then, is really an expansion of the number scale so that all numbers of the polyadic system within the unique (i.e., useful) range of the residue number system are units digits. Then, since all numbers of interest are units digits, the notions of carry and borrow are meaningless for arithmetic operations in residue number systems.

Finally, since an arbitrary set of moduli constitutes the basis of a residue number system, there are an infinite number of possible systems.

III. The Residue-to-Decimal Conversion Problem

Problem Description

The problem of residue-to-decimal number conversion can be reduced to three more fundamental problems, all of which are common to general-purpose digital computer applications of any number system.

It was shown in Section II that every residue number system contains only M distinct residue number representations, where M is the product of the (pairwise prime) moduli. Any operation which exceeds the range of unique representations, therefore, exceeds the range of unique conversion capability. This is the problem of capacity.

Any number system, to be useful in computer applications, must possess the capability of representing negative numbers. Since residue number systems possess only zero-order positional significance, any part of the unique range of representation of a residue number system can be allocated for the representation of negative numbers. Once the range of negative number representations has been assigned, however, some means must be readily available to determine whether any given number represented by the system is positive or negative. This is the problem of algebraic sign determination.

Finally, for computer results to be useful, some means must be available to make them intelligible to a human operator. This is the problem of machine language to human

language translation.

Problem Analysis

There are two immediate solutions to the problem of capacity, the first of which results in an apparant paradox, and the second of which shifts responsibility for the problem from the device to its human operator.

Since every residue number system consists of the residue-coded units digits of an M-adic number system, if the residues of the n (pairwise prime) moduli m_1 , m_2 , ... , m_n be denoted by r_1 , r_2 , ... , r_n , any decimal number x can be expressed as

$$x = M\left[\frac{x}{M}\right] + (r_1 , r_2 , \ldots , r_n)$$

Then, if the value of $\left[\frac{x}{M}\right]$ could be first determined, and then operated upon through arithmetic operation upon the residue numbers of the system, the unique range of the system could be extended without limit.

It was shown in Section II, however, that the above representation of x is really a polyadic representation of the form

$$x = a_o + a_1M + a_2M^2 + a_3M^3 + \ldots + a_nM^n$$

where

$$a_o = (r_1 , r_2 , \ldots , r_n)$$

and

$$\left[\frac{X}{M}\right] = a_1 + a_2M + a_3M^2 + \ldots + a_nM^{n-1}$$

so that the most important feature of a residue number system (i.e., effective expansion of the number scale to permit all numbers of interest to be expressed as units digits, thereby eliminating the need for carry and borrow in arithmetic operations) is destroyed.

The second solution to the problem of capacity takes cognizance of the fact that all digital computers are capacity limited. An obvious solution, then, is simply to make M, the number of unique representations, larger than the largest number of interest for the individual computer by increasing the magnitude and/or the number of the (pairwise prime) moduli. The problem of capacity then becomes the responsibility of the computer programmer. This imposes no greater limit on the generality of computer application than that already imposed upon existing computers.

The algebraic sign determination problem can be solved if variations in the ranges of the residue number system allocated to represent negative numbers are limited to increments of one of the moduli, which will be defined as m_L , the limiting modulus. The sign of any number can then be determined by comparing the incremental range within which the number falls to the ranges in which numbers are negative.

This limitation on break-point location between positive and negative numbers on the residue number scale does not impose a serious restriction upon the generality of computer application since any modulus of any residue number system is small compared with the range of unique representations. Furthermore, the maximum displacement of a possible break-point from some specifically desired break-point is $\frac{1}{2}m_L$, where m_L is the limiting modulus.

The machine language to human language translation problem can be solved if the smallest modulus of the residue number system is chosen as the above-mentioned limiting modulus. It will be shown in Section IV that this selection results in unique residue combinations of the remaining moduli in each of the incremental ranges, by which the decimal number represented by the residue number can be determined through inter- and intra-range comparisons. The logic required to implement this decimal number determination is greatly reduced if the limiting modulus is required to be less than, or equal to, half of ten, the base of the decimal number system.

These restrictions impose no limit on the generality of computer application, because the only requirement on the moduli of a residue number system is that they be relatively prime, and this includes an infinite set of numbers. Furthermore, selection of the least modulus as the limiting modulus reduces the limitations imposed by the sign determination solution.

IV. Residue-to-Decimal Conversion

The solution to the residue-to-decimal conversion problem described below is a general one which may be applied to any residue number system. For clarity of explanation, however, the solution is given in terms of the 3-4-5 residue number system, and is related to the relay device constructed to demonstrate a practical implementation of the principles involved.

Of the two solutions to the capacity problem discussed in Section III, the unlimited extension of the unqiue range of residue number representations through determination of, and operation upon, $\left[\frac{X}{M}\right]$ offers, by far, the greater challenge to the imagination. On the other hand, the extension of the unique range of residue number representations through an increase in the value of M offers an immediately practicable solution, if not necessarily the more efficient one. The capacity problem, therefore, is assumed to be solved by making M greater than the largest number of interest, or, restated in terms of a fixed M, by allowing interest in only those numbers which fall within the range of unique representation.

Solutions to the two remaining problems associated with residue-to-decimal conversion (i.e., sign determination, and machine language to human language translation) are provided by the four basic stages comprising the residue-to-decimal

converter: The Mod 3 Residue Reducing Stage; the Tens Digit Determining Stage; the Units Digit Determining Stage; and the Range and Sign Determining Stage.

Mod 3 Residue Reducing Stage

Let 3, the smallest modulus of the 3-4-5 residue number system, be selected as m_L, the limiting modulus. Then the function of the Mod 3 Residue Reducing Stage is to reduce an input residue number to a Mod 3 Base Residue Number, where a Mod 3 Base Residue Number is defined as any residue number in which the Mod 3 residue is zero; and Mod m Base Residues are defined as those residues which appear in Mod 3 Base Residue Numbers.

Referring to Table 2, for example, (0, 0, 0) is the Mod 3 Base Residue Number of the residue set (0, 0, 0), (1, 1, 1), and (2, 2, 2); (0, 3, 3) is the base number of the residue set (0, 3, 3), (1, 0, 4), and (2, 1, 0); (0, 2, 1) is the base number of the set (0, 2, 1), (1, 3, 2), and (2, 0, 3); and (0, 1, 4) is the base number of the set (0, 1, 4), (1, 2, 0), and (2, 3, 1). Furthermore, the Mod 4 and Mod 5 Base Residues of the four Mod 3 Base Residue Numbers indicated are (0, 0), (3, 3), (2, 1), and (1, 4), respectively. The Mod 3 Base Residue of a Mod 3 Base Residue Number is necessarily zero.

Reduction to base residue numbers of an entire residue number system with moduli m_1, m_2, ..., m_n, where $m_1 = m_L$, results in an interior residue number system with the modulus

$m_1 = 3$	$m_2 = 4$	$m_3 = 5$
0	0	0
1	1	1
2	2	2
0	3	3
1	0	4
2	1	0
0	2	1
1	3	2
2	0	3
0	1	4
1	2	0
2	3	1

Table 2

Partial Residue Number Table for Moduli 3-4-5

to demonstrate the concepts of

Mod 3 Base Residue Numbers

and Mod m Base Residues.

m_1 eliminated, and where the

$$\hat{m}_1 = \frac{1}{m_1} \prod_{i=0}^{n-1} m_i$$

base residue numbers are uniquely determined by the base numbers of the original system. Thus, in Table 3, for moduli 3-4-5 there are twenty alphabetically identifiable Mod 3 Base Residue Numbers.

An examination of Table 3 reveals that the Mod m Base Residues of any number can be determined by subtracting the Mod 3 residue from each of the residue of the number. Tables 4 and 5 represent the determinations of the Mod 4 and Mod 5 Base Residues, respectively, of Mod 3 Base Residue Numbers. The actual reductions of Mod 4 and Mod 5 residues to Mod 4 and Mod 5 Base Residue, respectively, can be accomplished by a switching-circuit implementation of Tables 4 and 5, as in Figure 1.

By virtue of the fact that whenever the Mod 3 residue is zero, the Mod 4 and Mod 5 residues are, by definition, Base Residues, however, a significant reduction in the equipment necessary to realize the switching functions of Figure 1 can be achieved. For example, Figure 2 is a wiring diagram of a 2-Relay method by which a Mod 4 residue of 3 (i.e., $|x|_4 = 3$) can be reduced to Mod 4 Base Residues.

If the Mod 3 residue is zero, both relays are deenergized, and the input signal appears on the output 3 line. If the Mod 3 residue is 1 or 2, relays 1 or 2 are energized,

Mod	3	4	5	Mod	3	4	5
Z	0	0	0	K	0	2	0
	1	1	1		1	3	1
	2	2	2		2	0	2
X	0	3	3	J	0	1	3
	1	0	4		1	2	4
	2	1	0		2	3	0
W	0	2	1	H	0	0	1
	1	3	2		1	1	2
	2	0	3		2	2	3
V	0	1	4	G	0	3	4
	1	2	0		1	0	0
	2	3	1		2	1	1
T	0	0	2	F	0	2	2
	1	1	3		1	3	3
	2	2	4		2	0	4
S	0	3	0	E	0	1	0
	1	0	1		1	2	1
	2	1	2		2	3	2
P	0	2	3	D	0	0	3
	1	3	4		1	1	4
	2	0	0		2	2	0
N	0	1	1	C	0	3	1
	1	2	2		1	0	2
	2	3	3		2	1	3
M	0	0	4	B	0	2	4
	1	1	0		1	3	0
	2	2	1		2	0	1
L	0	3	2	A	0	1	2
	1	0	3		1	2	3
	2	1	4		2	3	4

Table 3

Residue Number Table to demonstrate

Alphabetic Identification of

Mod 3 Base Residue Numbers.

Mod 4 Residues	Mod 3 Residues 0	1	2
0	0	3	2
1	1	0	3
2	2	1	0
3	3	2	1

(Body entries: Mod 4 Base Residues)

Table 4

Determination of

Mod 4 Base Residues

Mod 5 Residues	Mod 3 Residues 0	1	2
0	0	4	3
1	1	0	4
2	2	1	0
3	3	2	1
4	4	3	2

Table 5

Determination of

Mod 5 Base Residues

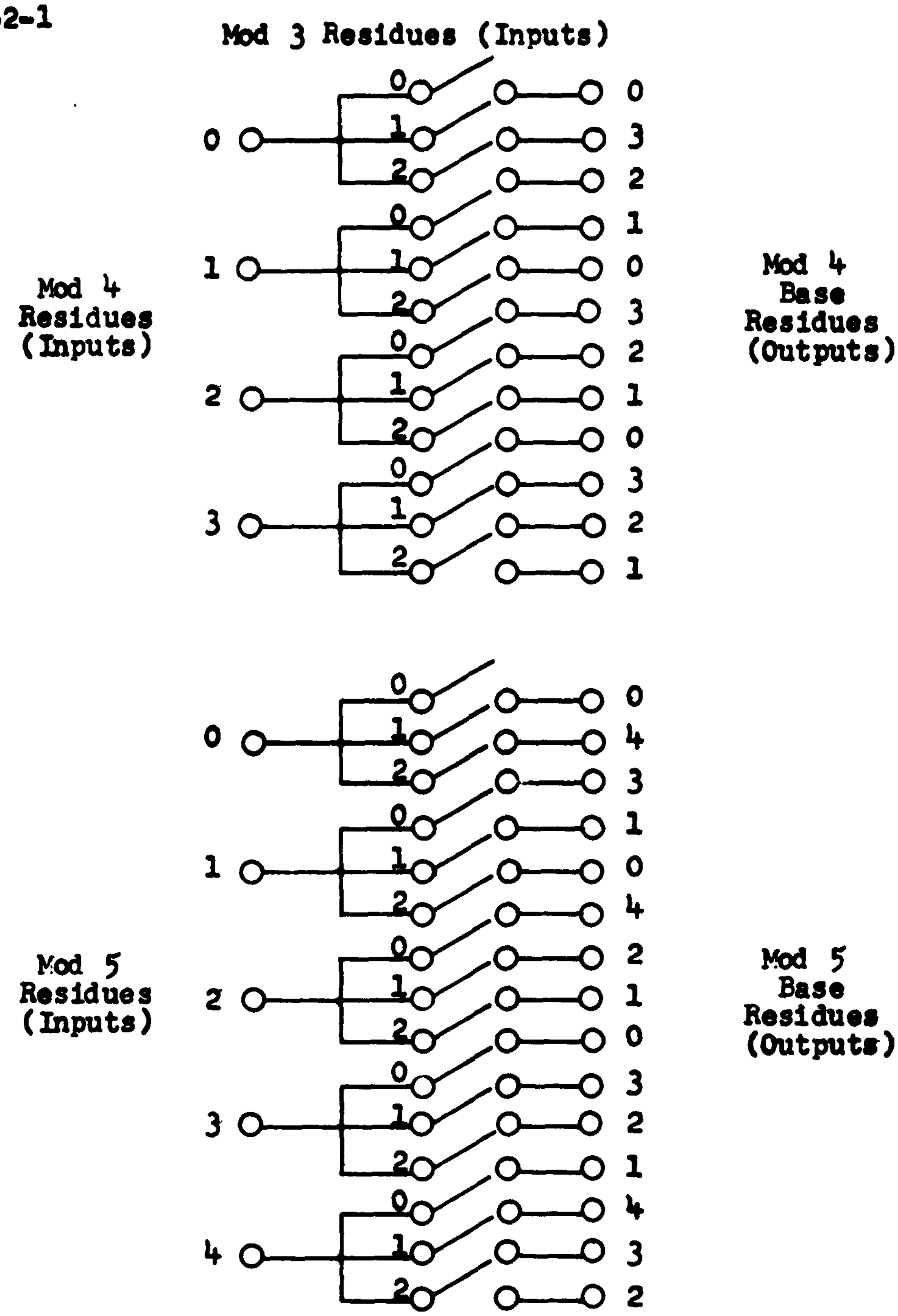


Figure 1

Switching Circuits to perform Reductions of Mod 4 and Mod 5 Residues to Mod 4 and Mod 5 Base Residues, respectively

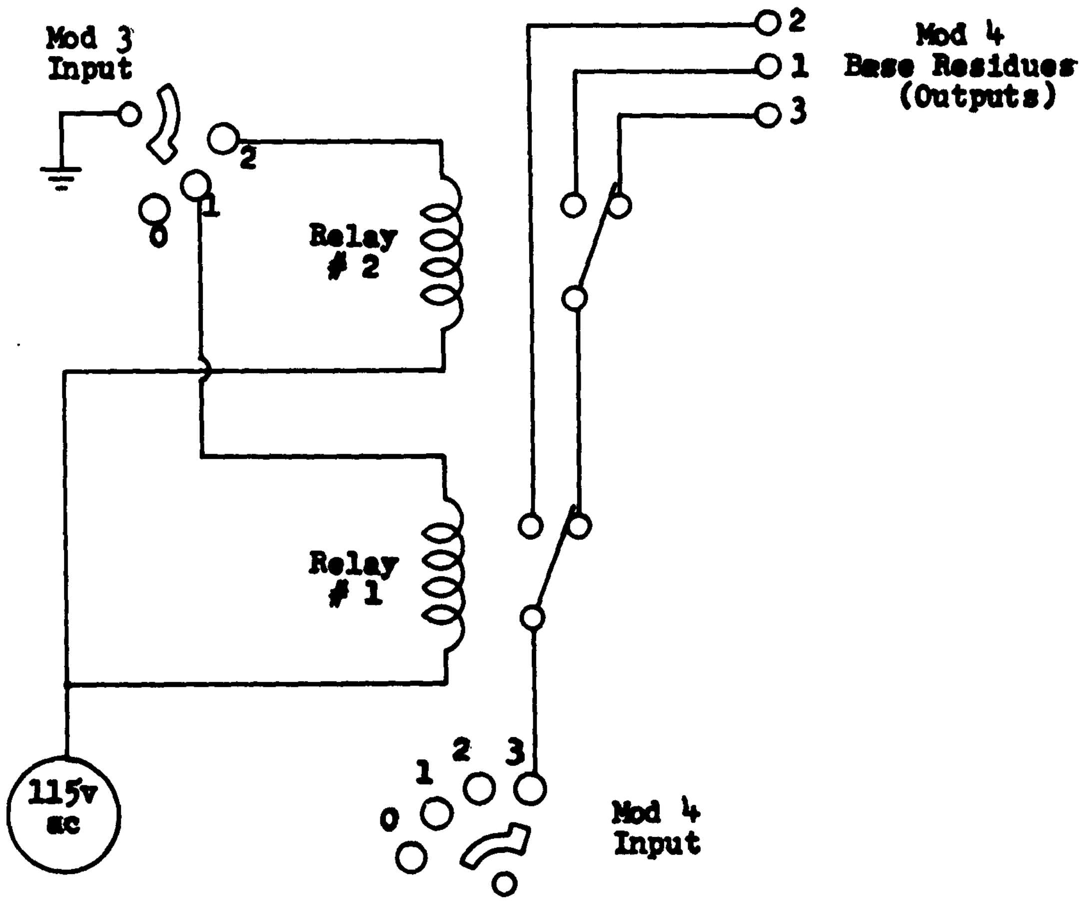


Figure 2

Wiring Diagram of a 2-Relay Method of Reduction of Mod 4 Residue of 3 to Mod 4 Base Residues

respectively, and the input signal appears on the 2 or 1 output line, respectively.

The wiring diagram of the complete Mod 3 Residue Reducing Circuit is shown in Figure 3.

Reduction of a residue number to its Mod 3 Base Residue Number effectively determines the incremental range of the limiting modulus within which the number lies, which is the necessary preliminary step in the solutions to the sign determination, and machine language to human language translation problems.

Tens Digit Determining Circuit

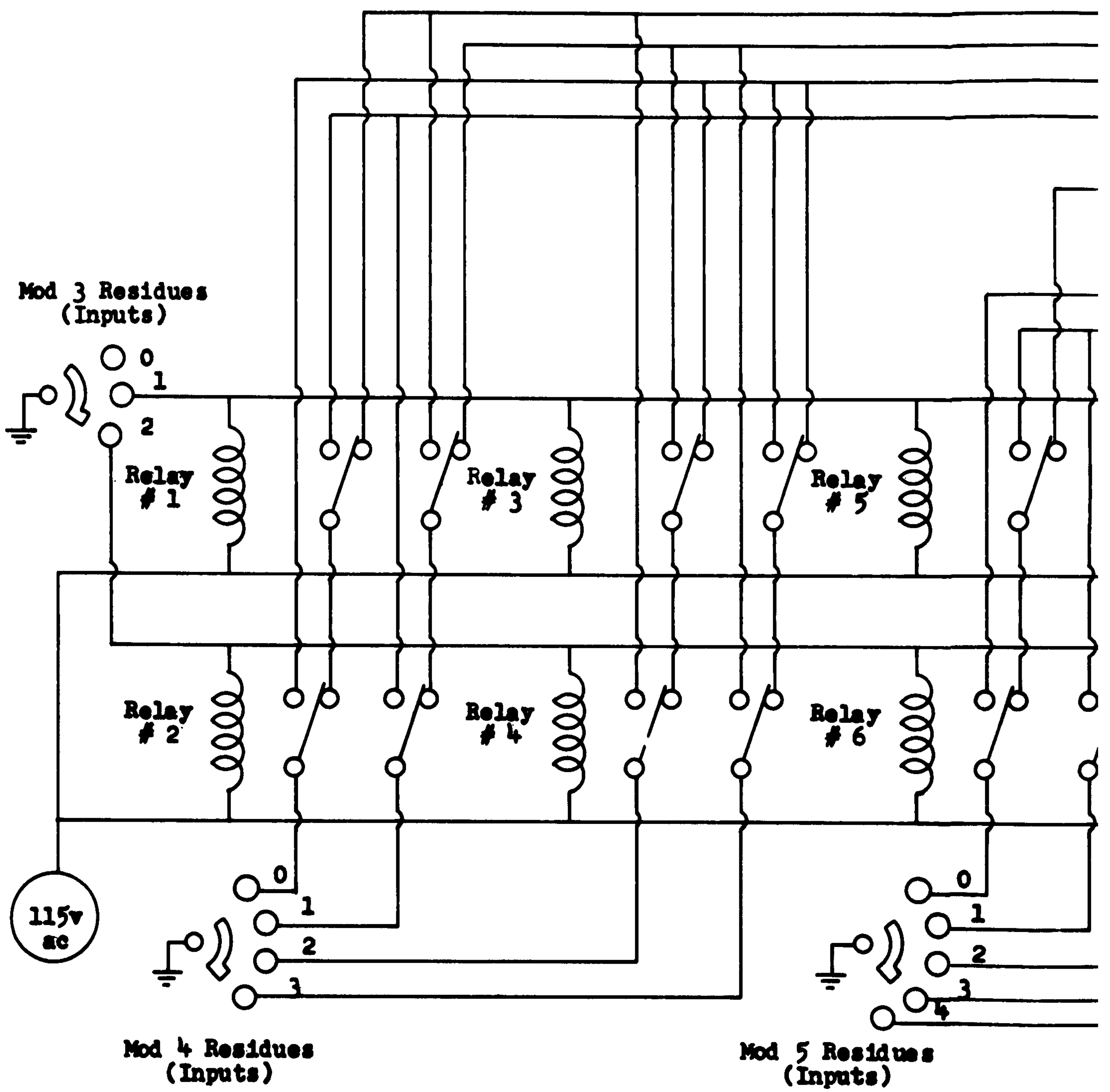
With the incremental range of the limiting modulus within which a residue number falls determined, a solution to the machine language to human language translation problem is readily available.

Table 6 is the decimal-residue number table for moduli 3-4-5, in the range of decimal numbers 0 to 59. From Tables 6 and 2, Table 7 can be extracted, showing the decimal equivalents of all Mod 3 Base Residue Numbers, with their Mod 4 and Mod 5 Base Residues, in the range of decimal numbers 0 to 59.

A comparison of Tables 6 and 7 reveals that each Mod 3 Base Residue Number represents a set of three residue numbers which reduce to that base number, and that, with the exceptions of base numbers V, P, G, and D, each base number represents three

GE/EE/62-1

1



Mod 3 Residues
(Inputs)
0
1
2
Relay # 1
Relay # 3
Relay # 5
Relay # 2
Relay # 4
Relay # 6
115v ac
0
1
2
3
Mod 4 Residues
(Inputs)
0
1
2
3
4
Mod 5 Residues
(Inputs)

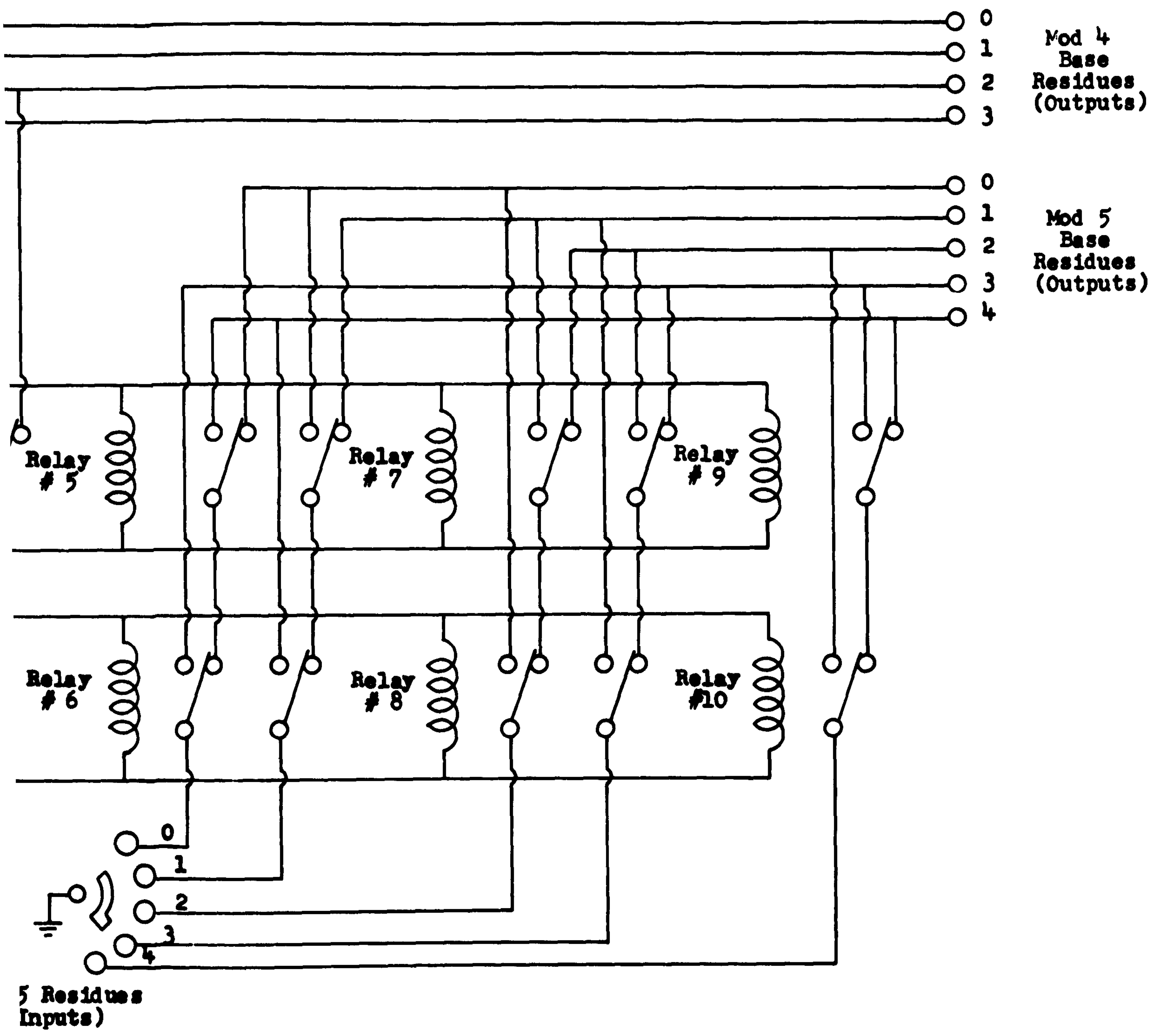


Figure 3

Mod 3 Residue Reducing Circuit

Decimal	Mod 3	Mod 4	Mod 5	Decimal	Mod 3	Mod 4	Mod 5
0	0	0	0	30	0	2	0
1	1	1	1	31	1	3	1
2	2	2	2	32	2	0	2
3	0	3	3	33	0	1	3
4	1	0	4	34	1	2	4
5	2	1	0	35	2	3	0
6	0	2	1	36	0	0	1
7	1	3	2	37	1	1	2
8	2	0	3	38	2	2	3
9	0	1	4	39	0	3	4
10	1	2	0	40	1	0	0
11	2	3	1	41	2	1	1
12	0	0	2	42	0	2	2
13	1	1	3	43	1	3	3
14	2	2	4	44	2	0	4
15	0	3	0	45	0	1	0
16	1	0	1	46	1	2	1
17	2	1	2	47	2	3	2
18	0	2	3	48	0	0	3
19	1	3	4	49	1	1	4
20	2	0	0	50	2	2	0
21	0	1	1	51	0	3	1
22	1	2	2	52	1	0	2
23	2	3	3	53	2	1	3
24	0	0	4	54	0	2	4
25	1	1	0	55	1	3	0
26	2	2	1	56	2	0	1
27	0	3	2	57	0	1	2
28	1	0	3	58	1	2	3
29	2	1	4	59	2	3	4

Table 6

Decimal-Residue Table for Moduli 3-4-5

in the Range of Decimal Numbers 0-59

Decimal	Mod 3	Mod 4	Mod 5	Decimal	Mod 3	Mod 4	Mod 5
0	Z	0	0	30	K	2	0
3	X	3	3	33	J	1	3
6	W	2	1	36	H	0	1
9	V	1	4	39	G	3	4
12	T	0	2	42	F	2	2
15	S	3	0	45	E	1	0
18	P	2	3	48	D	0	3
21	N	1	1	51	C	3	1
24	M	0	4	54	B	2	4
27	L	3	2	57	A	1	2

Table 7

Decimal Equivalents of all Mod 3 Base Residue Numbers

with their Mod 4 and Mod 5 Base Residues

in the Range of Decimal Numbers 0-59

Mod 5 Base Residues

Mod 4 Base Residues / **Tens Digits**

	0	1	2	3	4
0	0	3	1	D	2
1	4	2	5	3	V
2	3	0	4	P	5
3	1	5	2	0	G

Table 8

Tens Digit Determination from

Mod 4 and Mod 5 Base Residues

in the Range of Decimal Numbers 0-59

decimal numbers with the same tens digit as those represented by the other residue numbers of the set. Thus, the residue representation of 14 reduces to base number T, which represents decimal numbers with a tens digit of 1. Similarly, the residue representation of 28 reduces to base number L, which represents decimal numbers with a tens digit of 2. Neglecting the four exceptions, V, P, G, and D, for the moment, Table 8 can be constructed to demonstrate tens digit determinations from Mod 4 and Mod 5 Base Residues in the range of decimal numbers 0 to 59.

Since the Mod 4 and Mod 5 Base Residues are available as outputs from the Mod 3 Residue Reducing Circuit, a switching-circuit implementation of Table 8 can be devised, as shown in Figure 4.

The ambiguities in the tens digit determinations for base numbers V, P, G, and D, by the method of Table 8 and Figure 4, can be resolved by observing in Table 9 that base numbers V and G determine tens digits 0 and 3, respectively, when the units digit is 9, and tens digits 1 and 4, respectively, when the units digit is 0 or 1. Similarly, base numbers P and D determine tens digits 1 and 4, respectively, when the units digit is 8, or 9, and tens digits 2 and 5, respectively, when the units digit is 0. A truth-table arrangement of these observations can be constructed as in Table 10.

Referring to Table 10, it can immediately be seen that base numbers V and G each determine one tens digit when the

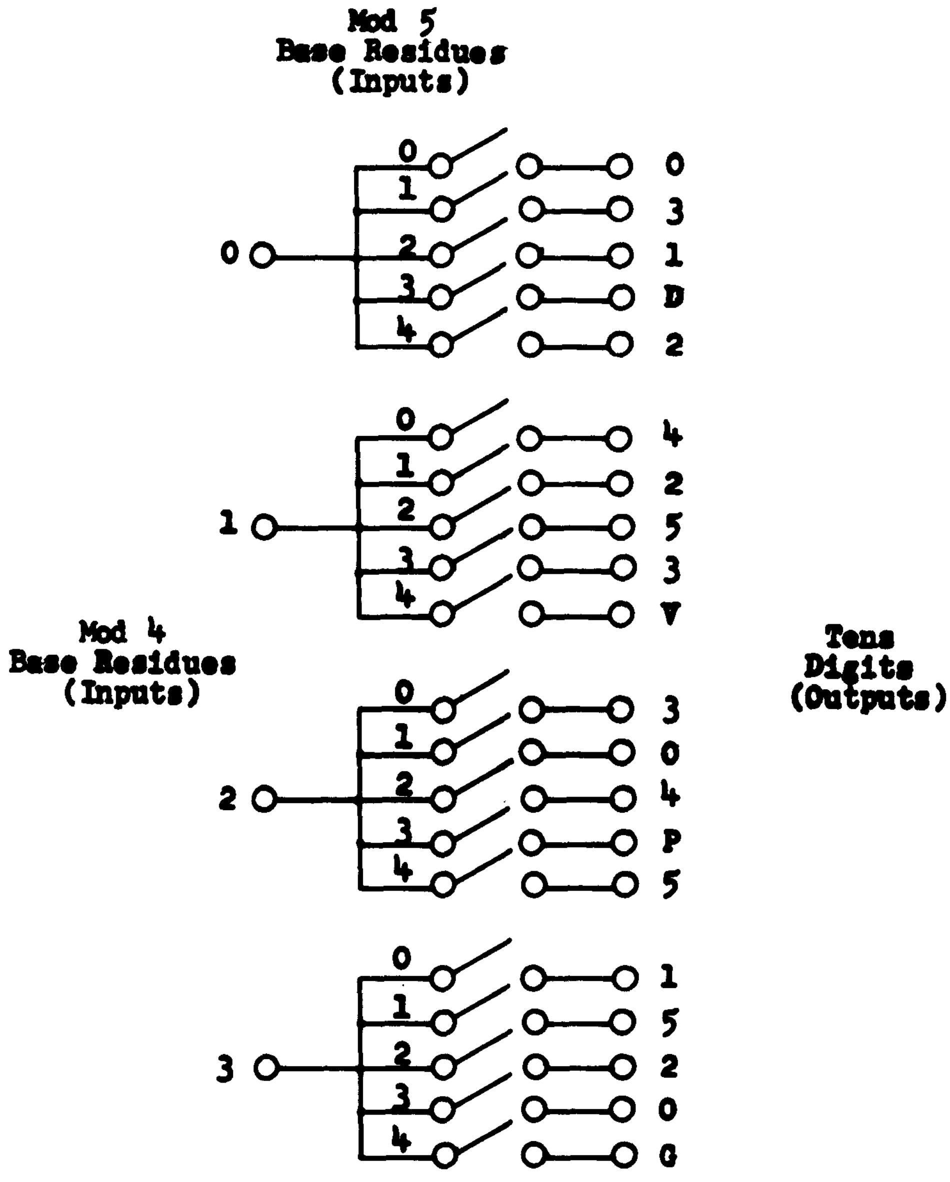


Figure 4

Switching-Circuit

Implementation of Table 8

Dec.	Base No.	Mod 3	Mod 4	Mod 5	Dec.	Base No.	Mod 3	Mod 4	Mod 5
.	.	.	.	.	.	.	.	.	.
9	V	0	1	4	39	G	0	3	4
10		1	2	0	40		1	0	0
11		2	3	1	41		2	1	1
.	.	.	.	.	.	.	.	.	.
18	P	0	2	3	48	D	0	0	3
19		1	3	4	49		1	1	4
20		2	0	0	50		2	2	0
.	.	.	.	.	.	.	.	.	.

Table 9

Partial Decimal-Residue Table

for Mod 3 Base Residue Numbers V, P, G, and D

Units Digits

Mod 3 Base Residue Number	8	9	0	1
V		0	1	1
G		3	4	4
P	1	1	2	
D	4	4	5	

Tens Digits

Table 10

Ambiguity Resolution of Tens Digit Determination

from Mod 3 Base Residue Numbers

by Units Digits Comparisons

units digit is 9, and another tens digit when the units digit is not 9; and base numbers P and D each determine one tens digit when the units digit is 0, and another tens digit when the units digit is not 0. Assuming that the units digits are available for comparison, these conditions can be realized by the switching-circuit of Figure 5.

The complete switching-circuit implementation of Tables 8 and 10, then, to provide unique tens digit determination in the range of decimal numbers 0 to 59 is as shown in Figure 6.

Figure 7 is the wiring diagram of a 3-relay realization of the switching functions required to uniquely determine the tens digit of a number whose residue number representation reduces to Mod 3 Base Residue Number D or P.

Both Mod 3 Base Residue Numbers D and P result in a Mod 5 Base Residue of 3, so either base number will energize relays 1 and 2. If the base number is D, a signal will appear on only the 0 Mod 4 Base Residue line. Then, if the units digit is not 0, the tens digit is 4, and if the units digit is 0, relay 3 is energized, and the tens digit is 5. Similarly, if the base number is P, a signal will appear on only the 2 Mod 4 Base Residue line. Then, if the units digit is not 0, the tens digit is 0, and if the units digit is 0, relay 3 is energized, and the tens digit is 1.

The wiring diagram of the complete Tens Digit Determining Circuit is shown in Figure 8.

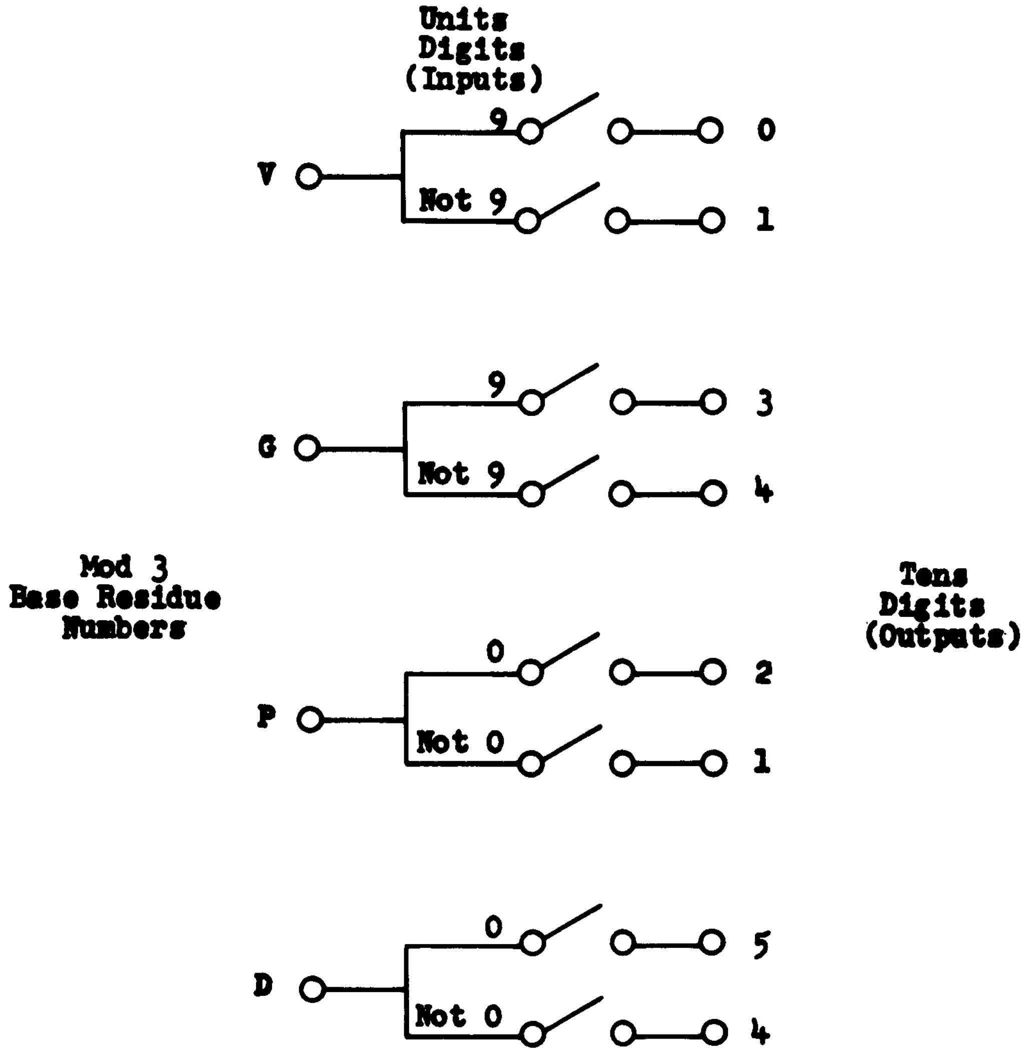


Figure 5

Switching-Circuit to Resolve

Tens Digit Determination Ambiguity

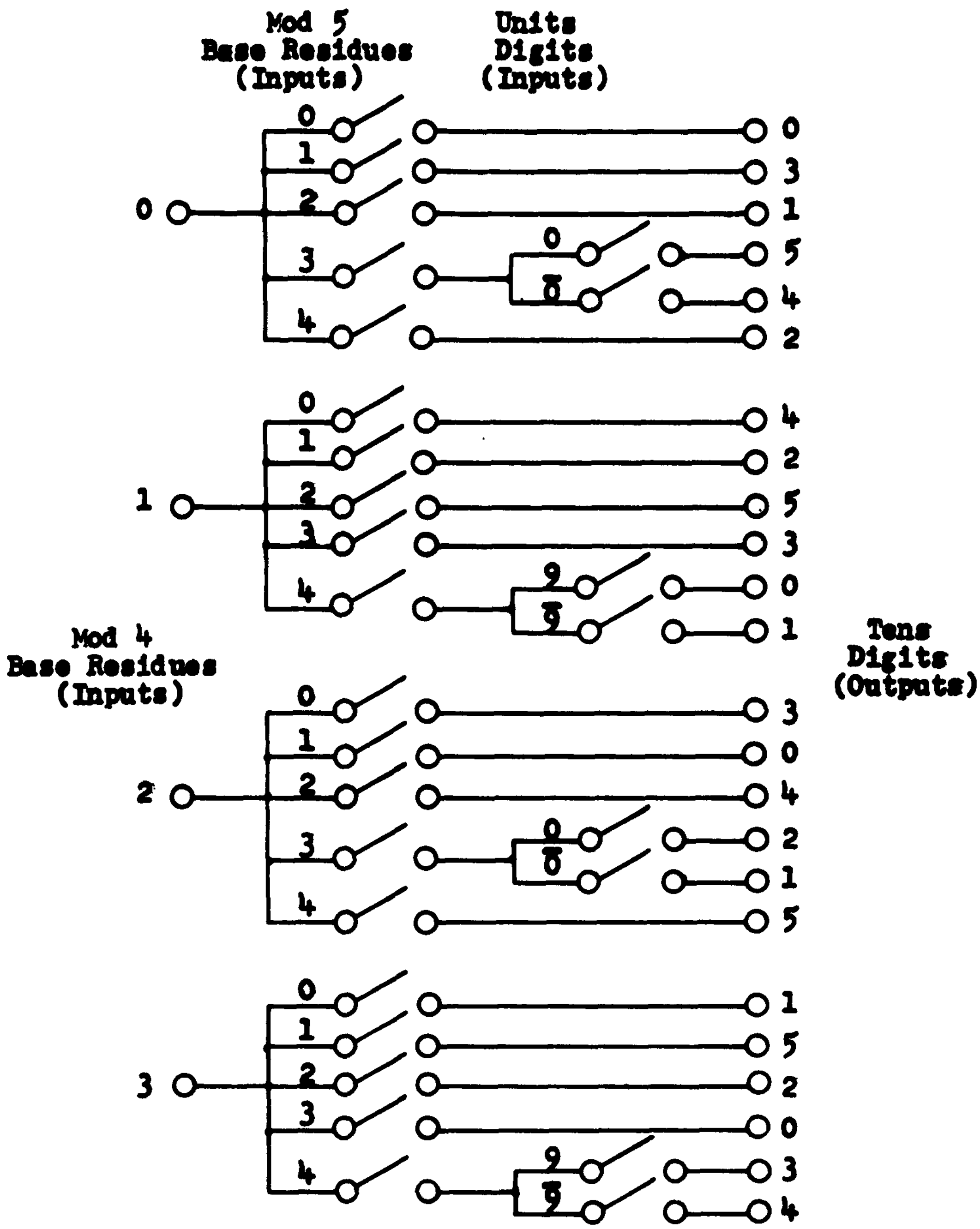


Figure 6

Switching-Circuit Implementation

of Tables 8 and 10

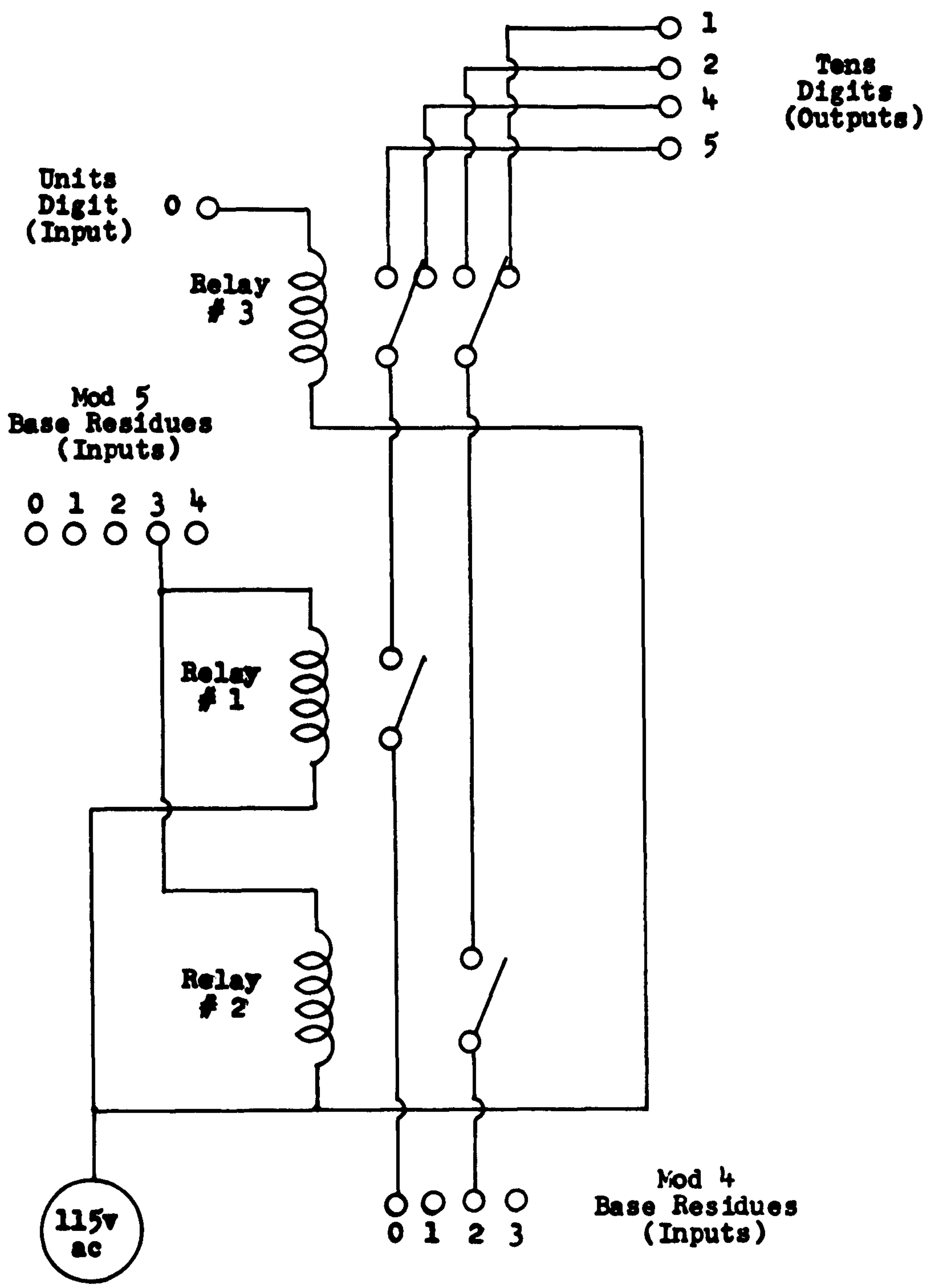
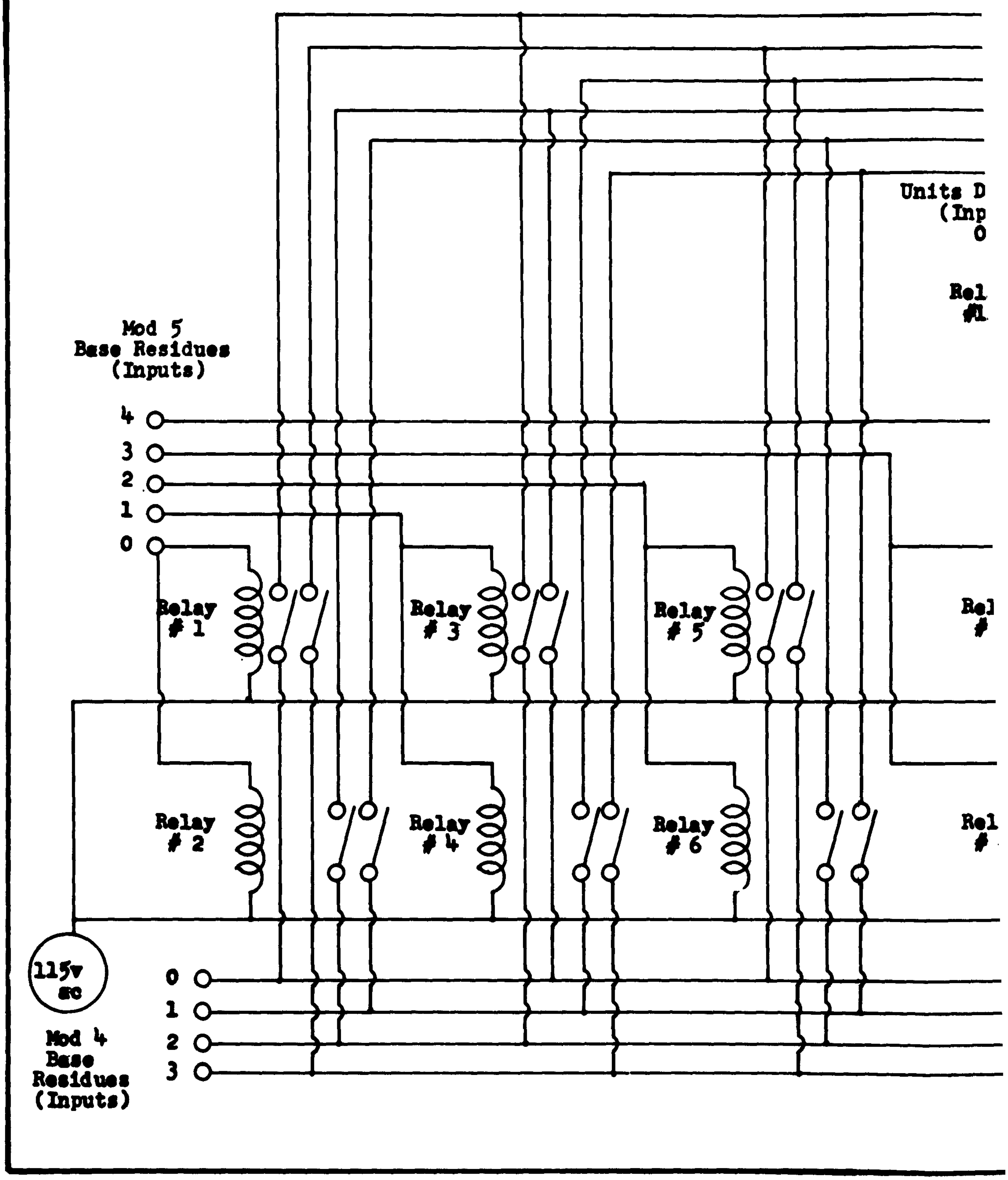


Figure 7

Wiring Diagram of a 3-Relay Method of

Tens Digit Determination of

Mod 3 Base Residue Numbers D or P


Units D
(Inp
0
Rel
#1
Mod 5
Base Residues
(Inputs)
4
3
2
1
0
Relay
1
Relay
3
Relay
5
Rel
#
Relay
2
Relay
4
Relay
6
Rel
#
115v
ac
0
1
2
3
Mod 4
Base
Residues
(Inputs)

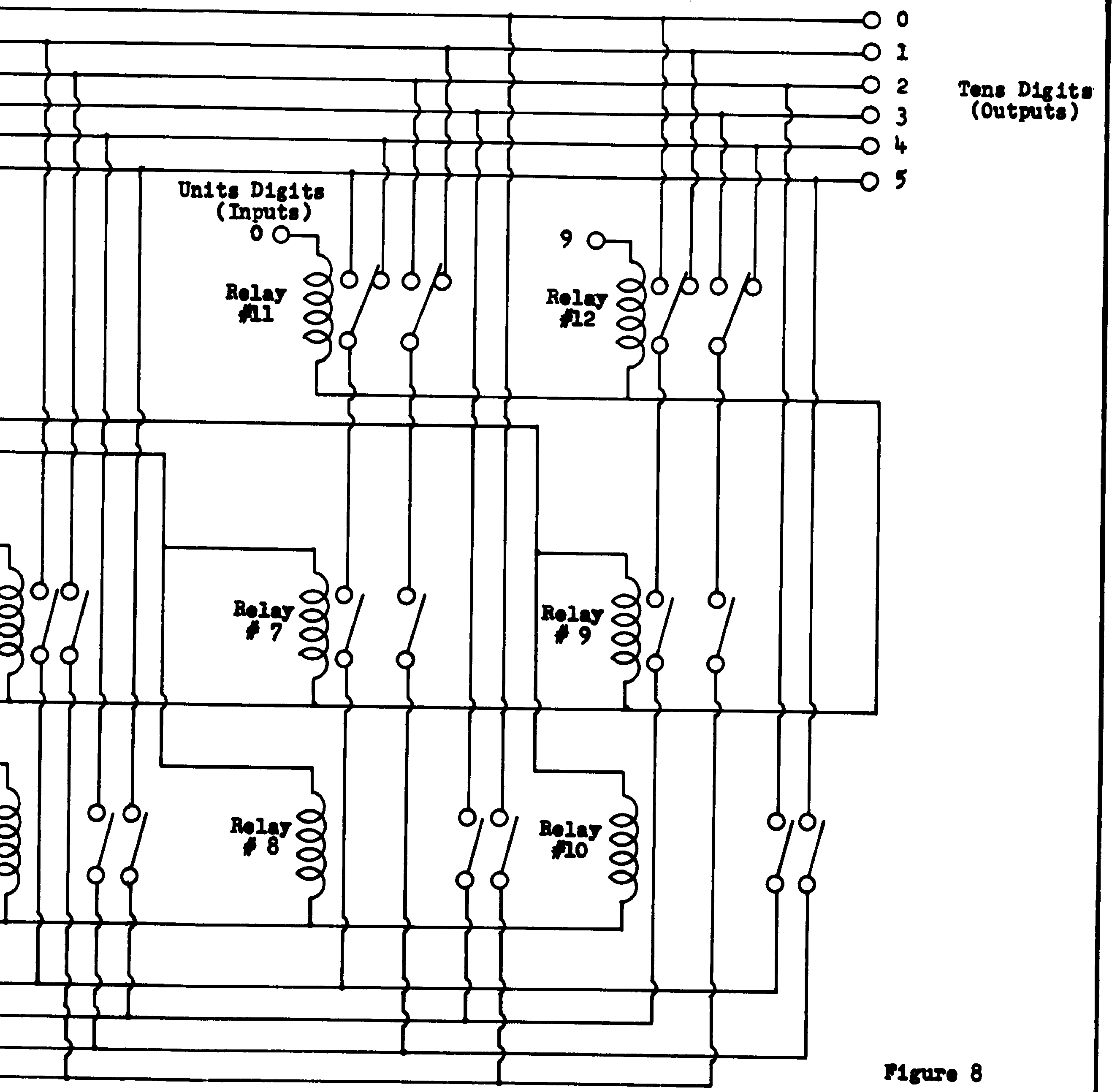


Figure 8

Tens Digit Determining Circuit
in the Range of Decimal Numbers 0-59

It is interesting to note that, although a resolution of the four ambiguous cases was accomplished through comparison of the units digit of the number, the resolution could also have been accomplished through a comparison of the Mod 3 residue of the number. Thus, in Table 9, base numbers V and G each determine one tens digit when the Mod 3 residue is 0, and another tens digit when the Mod 3 residue is not 0. Similarly, base numbers P and D each determine one tens digit when the Mod 3 residue is 2, and another tens digit when the Mod 3 residue is not 2. The circuitry necessary to duplicate this logic is identical to that actually employed in Figures 7 and 8.

Units Digit Determining Circuit

A method of units digit determination based on a comparison of the incremental range of the limiting modulus within which a number falls similar to that used in the Tens Digit Determining Circuit can be developed as follows:

Table 11 shows the Mod 3 residue equivalents and the Mod 3 Base Residue Numbers of all decimal numbers in the range 0 to 59. Figure 9 is a switching-circuit implementation of Table 11. A circuit can easily be constructed to duplicate the switching functions of Figure 9 to provide unique units digit determinations for all residue numbers in the range of decimal numbers 0 to 59.

In the interest of economy of equipment, however, a more

Decimal	Base Number	Mod 3	Decimal	Base Number	Mod 3
0	Z	0	30	K	0
1		1	31		1
2		2	32		2
3	X	0	33	J	0
4		1	34		1
5		2	35		2
6	W	0	36	H	0
7		1	37		1
8		2	38		2
9	V	0	39	G	0
10		1	40		1
11		2	41		2
12	T	0	42	F	0
13		1	43		1
14		2	44		2
15	S	0	45	E	0
16		1	46		1
17		2	47		2
18	P	0	48	D	0
19		1	49		1
20		2	50		2
21	N	0	51	C	0
22		1	52		1
23		2	53		2
24	M	0	54	B	0
25		1	55		1
26		2	56		2
27	L	0	57	A	0
28		1	58		1
29		2	59		2

Table 11

Mod 3 Residue-Decimal Table

in the Range of Decimal Numbers 0-59

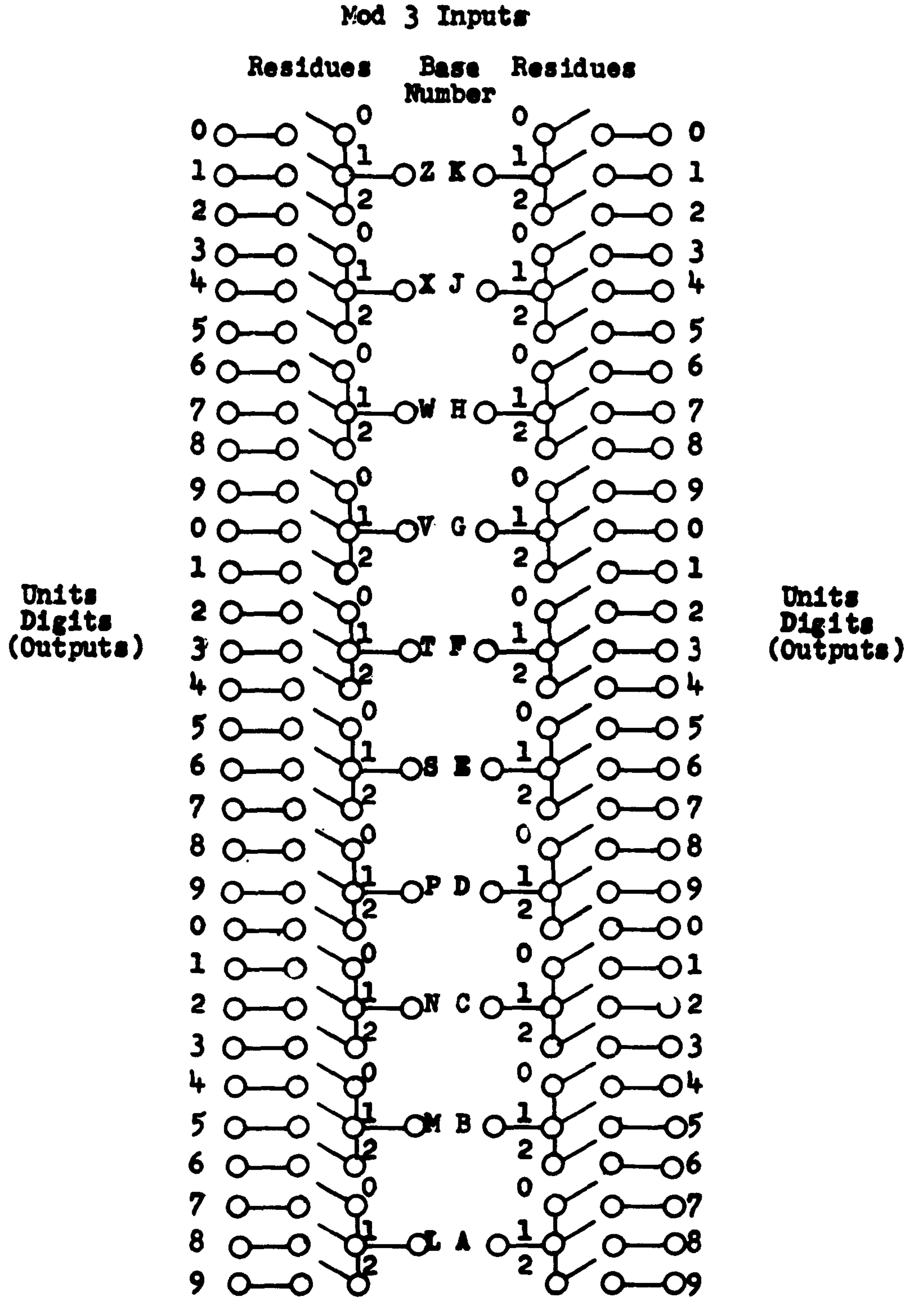


Figure 9

Switching-Circuit

Implementation of Table 11

direct approach to units digit determination was taken, based on a characteristic common to all residue number systems with one modulus m_{2p} some multiple of 2, and one modulus m_{5q} some multiple of 5.

The number of unique representations M of every residue number system with moduli m_1 , m_2 , ... , m_{2p} , m_{5q} , ... , m_n is some multiple t of 10, where

$$t = pq \prod_{i=1}^{n} \frac{m_i}{m_{2p} m_{5q}}$$

Consider the residue number system with moduli 2 and 5. Here p = q = 1, and t = 1. Then M = 10, and as demonstrated in Section II, each of the ten residue numbers uniquely determines one decimal number in the range 0 to 9, which is exactly the range of units digits of the decimal number system. Table 12 is a residue-decimal table for the 2-5 Residue Number System.

It was also demonstrated in Section II that any number x can be expressed as

$$x = m\left[\frac{x}{m}\right] + |x|_m$$

where

$$m\left[\frac{x}{m}\right] = m \sum_{i=1}^{n-1} a_{(i+1)} m^i$$

and

$$|x|_m = a_o$$

Mod 2 Residues	Mod 5 Residues: 0	1	2	3	4
0	0	6	2	8	4
1	5	1	7	3	9

Decimal Equivalents

Table 12

2-5 Residue Number System

Residue-to-Decimal Number Table

Mod 2 Residues	Mod 4 Residues		Mod 5 Residues: 0	1	2	3	4
0	0	2	0	6	2	8	4
1	1	3	5	1	7	3	9

Decimal Equivalents

Table 13

2-5 Residue Number System

Residue-to-Decimal Number Table

to Demonstrate the Distribution of

Mod 4 Residues Reduced to Mod 2 Residues

where a_o is the units digit of x in the m-adic number system.

Now, if m = 10t,

$$x = 10t \sum_{i=0}^{n-1} a_{(i+1)}(10t)^i + a_o$$

Then, applying properties associated with the reduction of a number to a least positive residue (Ref 1:1-2)

$$|x|_b = \left| 10t \sum_{i=0}^{n-1} a_{(i+1)}(10t)^i + a_o \right|_b$$

$$= \left| \, |10t|_b \sum_{i=0}^{n-1} a_{(i+1)}(10t)^i + a_o \right|_b$$

Now, if b is a divisor of 10, then $|10t|_b = 0$, and

$$|x|_b = |a_o|_b$$

Then, with a_o the units digit of a number when expressed in a polyadic number system when the radix is some multiple t of 10,

$$|x|_2 = |a_o|_2$$

$$|x|_5 = |a_o|_5$$

Furthermore, going back to the expression for x,

$$x = m\left[\frac{x}{m}\right] + |x|_m$$

if m is taken as the product of two factors, p and q, $m = pq$, and

$$x = pq\left[\frac{x}{pq}\right] + |x|_{pq}$$

Then

$$|x|_p = \left| pq\left[\frac{x}{pq}\right] + |x|_{pq} \right|_p$$

$$= \left| |pq|_p \left[\frac{x}{pq}\right] + |x|_{pq} \right|_p$$

But $|pq|_p = 0$, so that

$$|x|_p = \left| |x|_{pq} \right|_p$$

Then

$$|x|_2 = \left| |x|_{2p} \right|_2$$

$$|x|_5 = \left| |x|_{5q} \right|_5$$

Therefore, the units digit of a decimal number represented in any residue number system containing moduli m_{2p} and m_{5q} can be determined by reducing the Mod m_{2p} and Mod m_{5q} residues to residues Mod 2 and Mod 5, respectively, and implementing Table 12.

Explicit reductions of the m_{2p} and m_{5q} residues to residues Mod 2 and Mod 5 are not required, however, since these reductions merely result in each residue Mod 2 representing

p residues Mod m_{2p} , and each residue Mod 5 representing q residues Mod m_{5q} . Thus, in the 3-4-5 residue number system, p = 2, and

$$\left|0\right|_2 = \left|\left|0\right|_4\right|_2 = \left|\left|2\right|_4\right|_2 = 0$$

$$\left|1\right|_2 = \left|\left|1\right|_4\right|_2 = \left|\left|3\right|_4\right|_2 = 1$$

Table 13 is a residue-decimal table for the 2-5 Residue Number System, demonstrating the distribution of Mod 4 residues in accordance with their reductions to Mod 2 residues. Then the available Mod 4 and Mod 5 residues can be utilized directly to implement a switching-circuit determination of the decimal units digits, as shown in Figure 10.

Figure 11 is a wiring diagram of a 2-relay method of duplicating the switching functions of Figure 10 to provide units digit determination for a 0 Mod 5 residue (i.e., $\left|x\right|_5 = 0$).

With a Mod 5 residue of 0, relays 1 and 2 are energized. Then, with a Mod 4 residue of either 0 or 2, the units digit is 0, and with a Mod 4 residue of either 1 or 3, the units digit is 5.

Figure 12 is the wiring diagram of the complete Units Digit Determining Circuit.

Range and Sign Determining Circuit

The function of the Range and Sign Determining Circuit is to determine whether the incremental range of the limiting

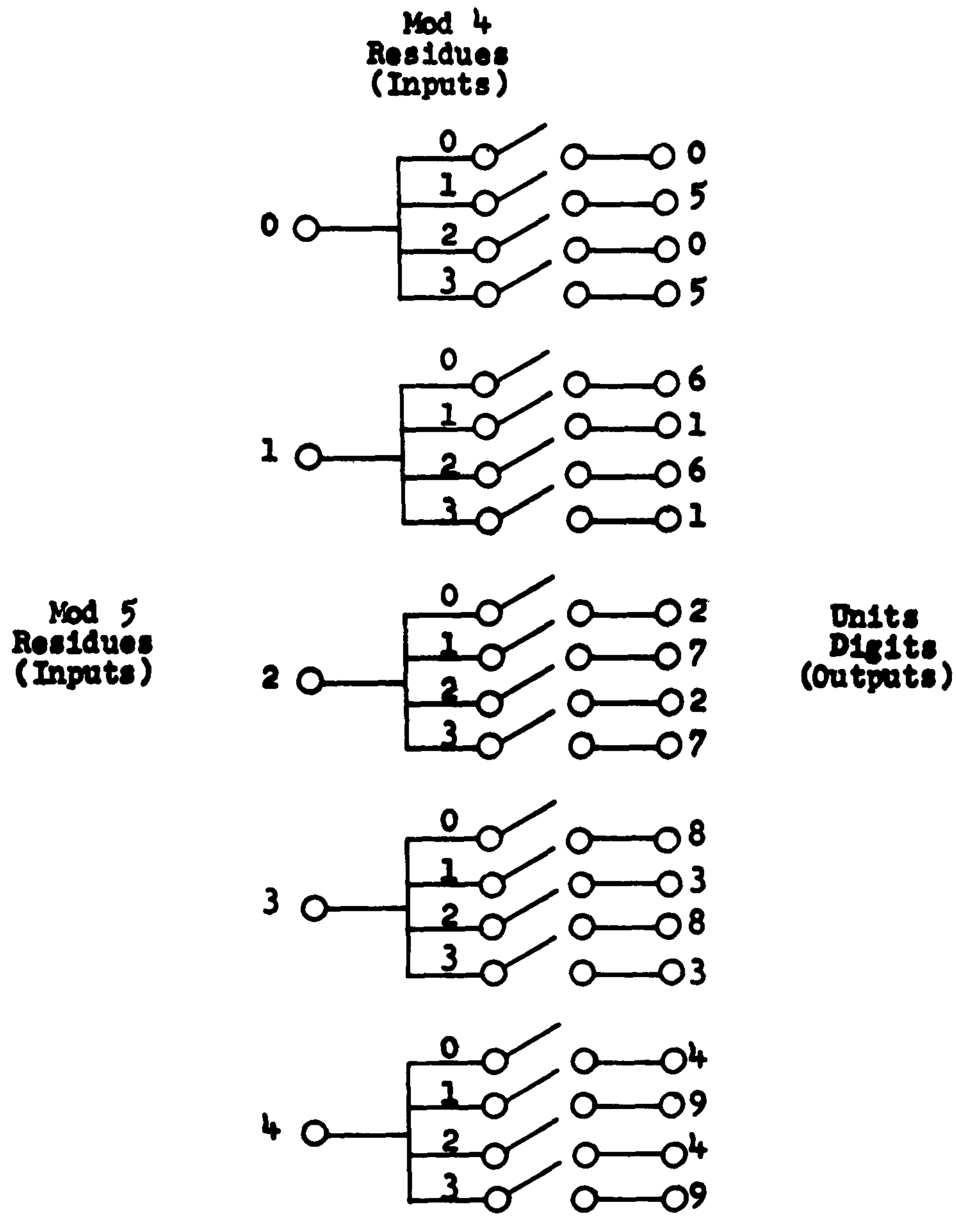


Figure 10

Switching-Circuit Implementation of Table 13

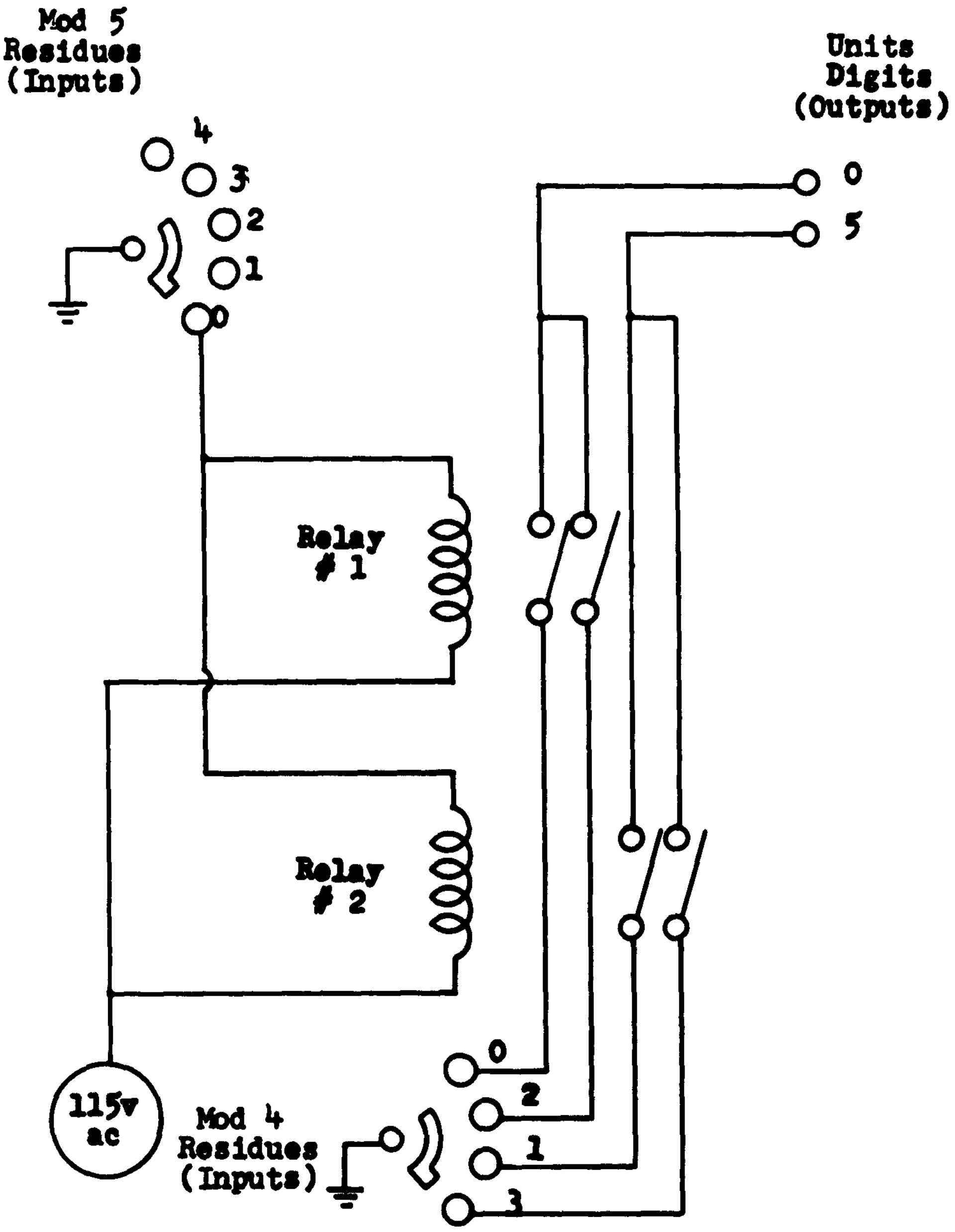
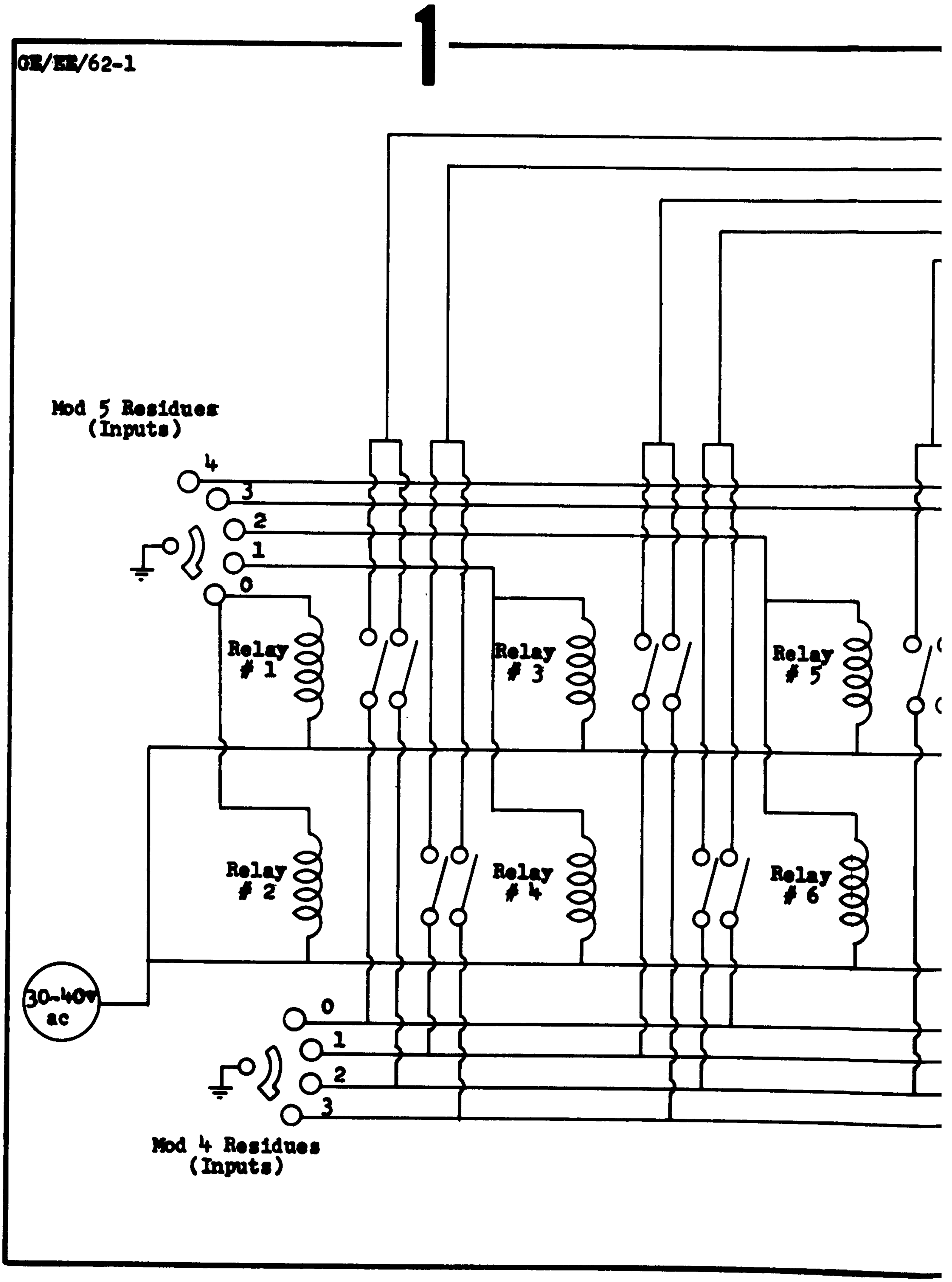


Figure 11

Units Digit Determination


1
Mod 5 Residues
(Inputs)
4
3
2
1
0
Relay
1
Relay
2
Relay
3
Relay
4
Relay
5
Relay
6
30-40v
ac
0
1
2
3
Mod 4 Residues
(Inputs)

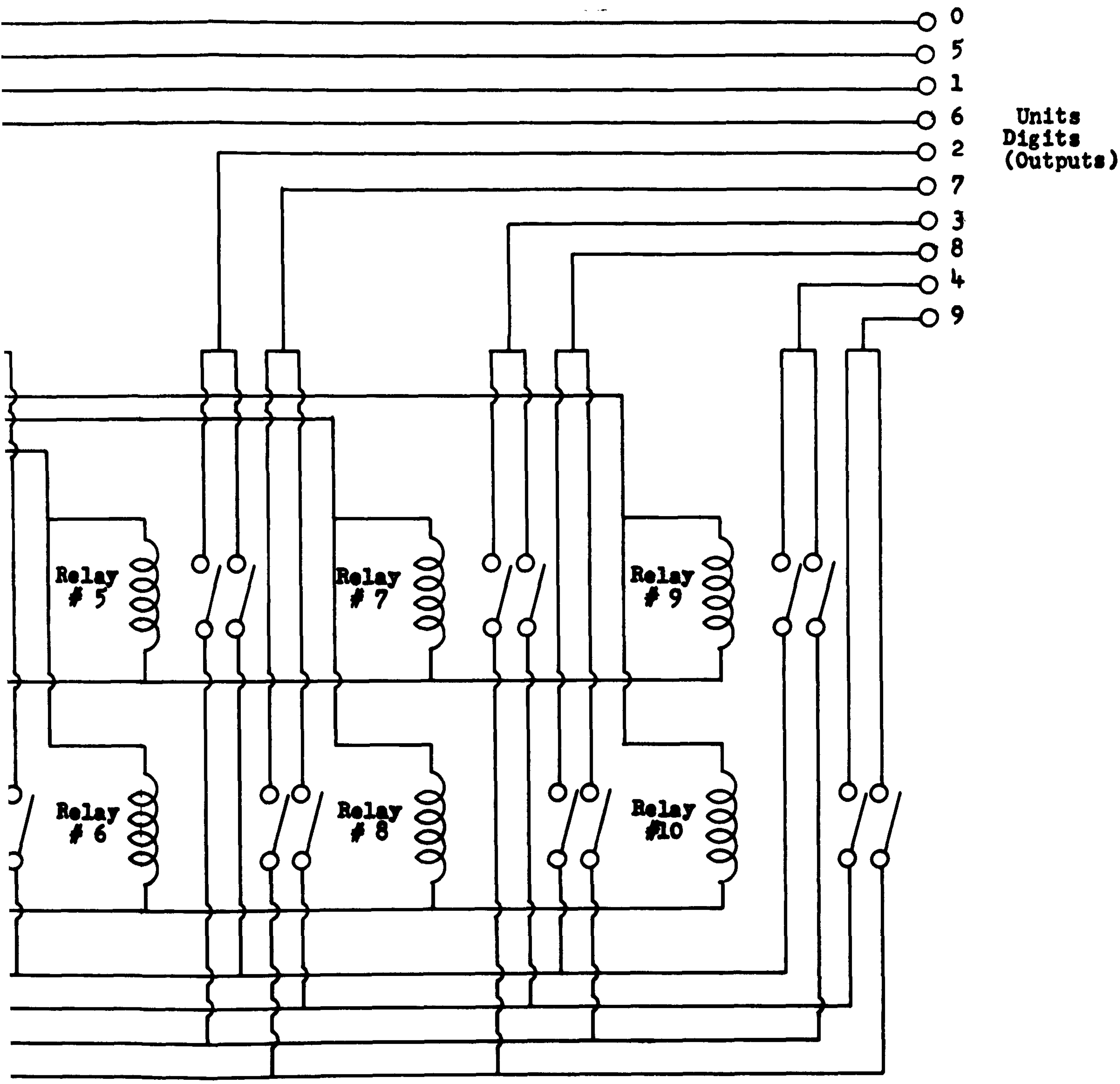


Figure 12

Units Determining Circuit

in the Range of Decimal Numbers 0-59

modulus within which a number falls is positive or negative, and to operate accordingly upon the output of the Tens and Units Digit Determining Circuits to produce the correct decimal equivalent of any residue number.

Referring to Table 7, it was demonstrated earlier that each Mod 3 Base Residue Number results in a unique combination of Mod 4 and Mod 5 Base Residues. Then, with the Mod 4 and Mod 5 Base Residues of any number available from the Residue Reducing Circuit, the particular base number to which any residue number reduces can be determined. Furthermore, since variations in the ranges of positive and negative numbers represented by the residue number system are limited to increments of 3, the limiting modulus, the sign of any number which reduces to a particular base residue number is necessarily identical to the sign of the base number.

Figure 12A is a switching-circuit implementation of Table 7, to determine the base number to which any residue number reduces. Then, since algebraic sign determination constitutes a binary decision, an additional switch at each base number output provides that decision.

Table 14 is a complete decimal-residue number table for all decimal numbers in the range -60 to 59. It can be seen in Table 14, that if base number A is negative, the range of decimal numbers represented by the residue number system lies between -3 and 56. Similarly, if both base numbers A and B are negative, the decimal range is -6 to 53. Then,

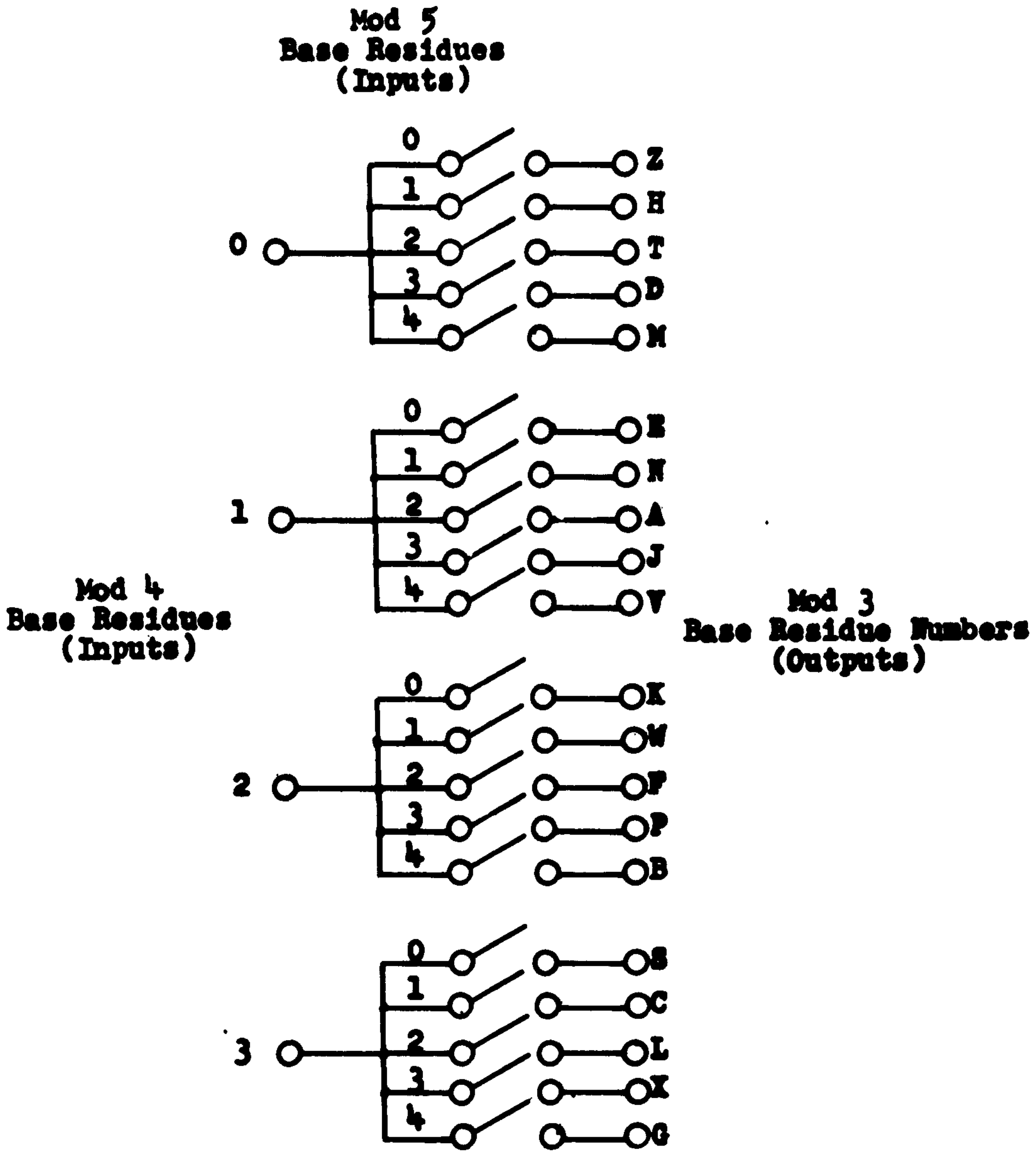


Figure 12A

Switching-Circuit Implementation

of Table 7

Decimal Pos	Decimal Neg	Base Number	Mod 3	Mod 4	Mod 5	Decimal Pos	Decimal Neg	Base Number	Mod 3	Mod 4	Mod 5
0	-60	Z	0	0	0	30	-30	K	0	2	0
1	-59		1	1	1	31	-29		1	3	1
2	-58		2	2	2	32	-28		2	0	2
3	-57	X	0	3	3	33	-27	J	0	1	3
4	-56		1	0	4	34	-26		1	2	4
5	-55		2	1	0	35	-25		2	3	0
6	-54	W	0	2	1	36	-24	H	0	0	1
7	-53		1	3	2	37	-23		1	1	2
8	-52		2	0	3	38	-22		2	2	3
9	-51	V	0	1	4	39	-21	G	0	3	4
10	-50		1	2	0	40	-20		1	0	0
11	-49		2	3	1	41	-19		2	1	1
12	-48	T	0	0	2	42	-18	F	0	2	2
13	-47		1	1	3	43	-17		1	3	3
14	-46		2	2	4	44	-16		2	0	4
15	-45	S	0	3	0	45	-15	E	0	1	0
16	-44		1	0	1	46	-14		1	2	1
17	-43		2	1	2	47	-13		2	3	2
18	-42	P	0	2	3	48	-12	D	0	0	3
19	-41		1	3	4	49	-11		1	1	4
20	-40		2	0	0	50	-10		2	2	0
21	-39	N	0	1	1	51	- 9	C	0	3	1
22	-38		1	2	2	52	- 8		1	0	2
23	-37		2	3	3	53	- 7		2	1	3
24	-36	M	0	0	4	54	- 6	B	0	2	4
25	-35		1	1	0	55	- 5		1	3	0
26	-34		2	2	1	56	- 4		2	0	1
27	-33	L	0	3	2	57	- 3	A	0	1	2
28	-32		1	0	3	58	- 2		1	2	3
29	-31		2	1	4	59	- 1		2	3	4

Table 14

Decimal-Residue Number Table for all Decimal Numbers

in the Range -60 to 59

the alphabetically successive transition, from positive to negative, of base numbers A through Z, results in variations in decimal range from -60 to -1, to 0 to 59, in increments of three. A switching-circuit implementation of this logic is shown in Figure 13, where a signal on the output line indicates a negative number, and the lack of a signal indicates a positive number.

Figure 14 demonstrates the circuity utilized to duplicate the switching functions of Figure 13. A Mod 4 Base Residue of 0 energizes relay 1. Then, with a Mod 5 Base Residue of 0, 1, 2, 3, or 4, ~~and~~ switches Z, H, T, D, or M closed, respectively, a (negative number) signal appears at the output. Otherwise, a (positive number) lack of a signal appears at the output.

Aiken and Semon show (Ref 1:1-5) that the residue Mod m of a negative number -x is the complement on m of the residue of x. Therefore, since the decimal numbers represented by the residues of the 3-4-5 system are, in reality, residues of numbers Mod 60, every positive decimal number converts to a negative number of magnitude (M - x) = 60 - x.

Then, since the range of unique representations M = 60 is a multiple of ten, the units digit of the negative of a number is the complement on 10 of the units digit of the number, as demonstrated in Table 15. (This will be true for any residue number system where one modulus is even, and another is some multiple of 5.) The tens digit of a negative

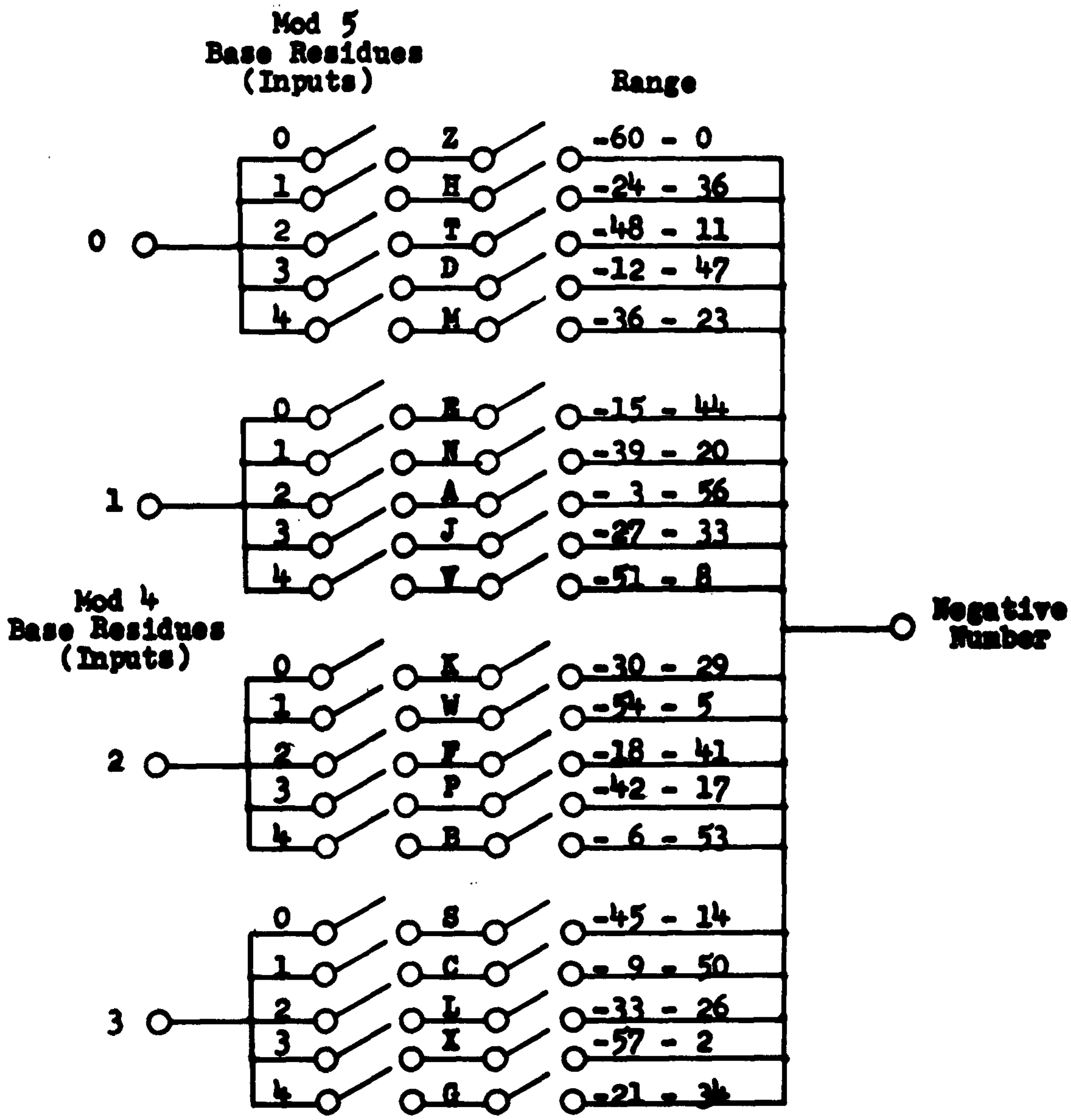


Figure 13

Switching-Circuit Implementation

of Table 14

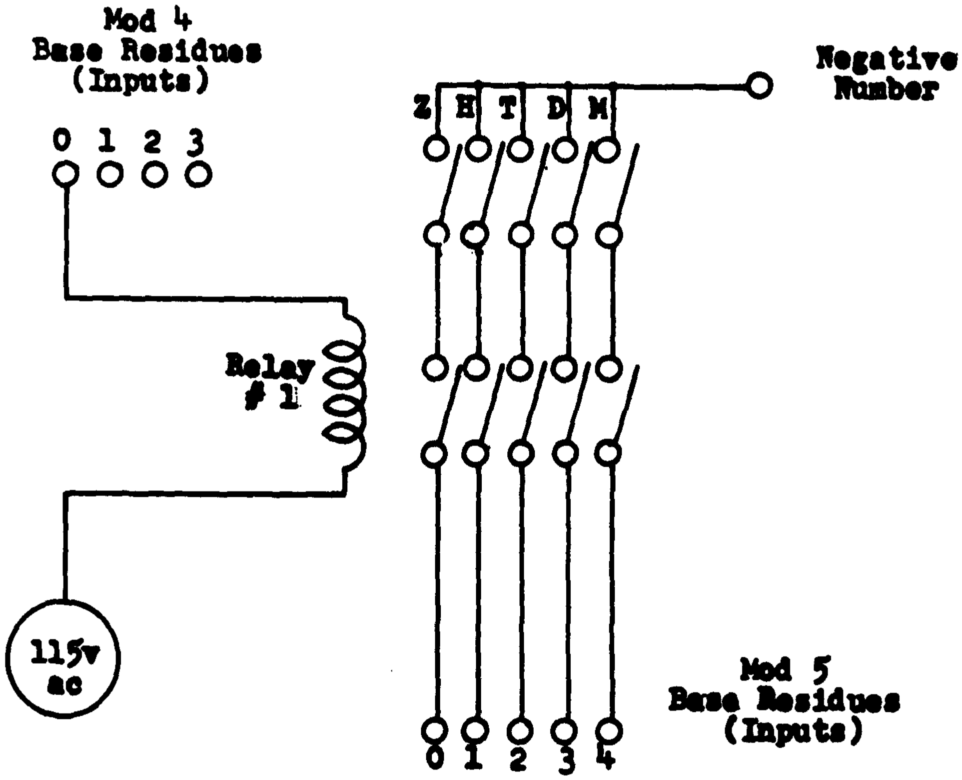


Figure 14

Wiring Diagram of Circuit

to Duplicate Switching Functions of Figure 13

Positive	Negative
0	0
1	9
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1

Table 15

Complemented Units Digits of

Positive and Negative Numbers

Positive	Negative	
	0	$\bar{0}$
0	6	5
1	5	4
2	4	3
3	3	2
4	2	1
5	1	0

Table 16

Complemented Tens Digits of Positive

and Negative Numbers Modified by Units Digit Value

number can then be found as the complement on the tens digit of M, or of (M - 10), according to whether the units digit of the number is 0 or not, as shown in Table 16.

A switching-circuit implementation of Tables 15 and 16 is shown in Figure 15, and Figure 16 is the wiring diagram of the circuitry employed to duplicate the switching functions of Figure 15.

If the tens digit and units digit equivalents of a residue number are each 1, and eleven is in the positive range of numbers, relays 1, 2, and 3 are deenergized, so that the tens digit and units digit outputs are both 1. If eleven is in the negative range of numbers, however, relays 1 and 3 will be energized, and the tens digit and units digit outputs will be 4 and 9, respectively, with a negative number indicator energized. Had the units digit been 0, the units digit output would have been 0 for both positive and negative numbers. Relay 2 would than have been energized, however, and the tens digit output for a negative number would have been 5.

The wiring diagram of the complete Range and Sign Determining Circuit is shown in Figure 17.

The Residue-to-Decimal Number Converter

Figure 18 is the wiring diagram of the complete Residue-to-Decimal Number Converter. The indicated residue number input (2, 2, 2) can be traced through the integrated system

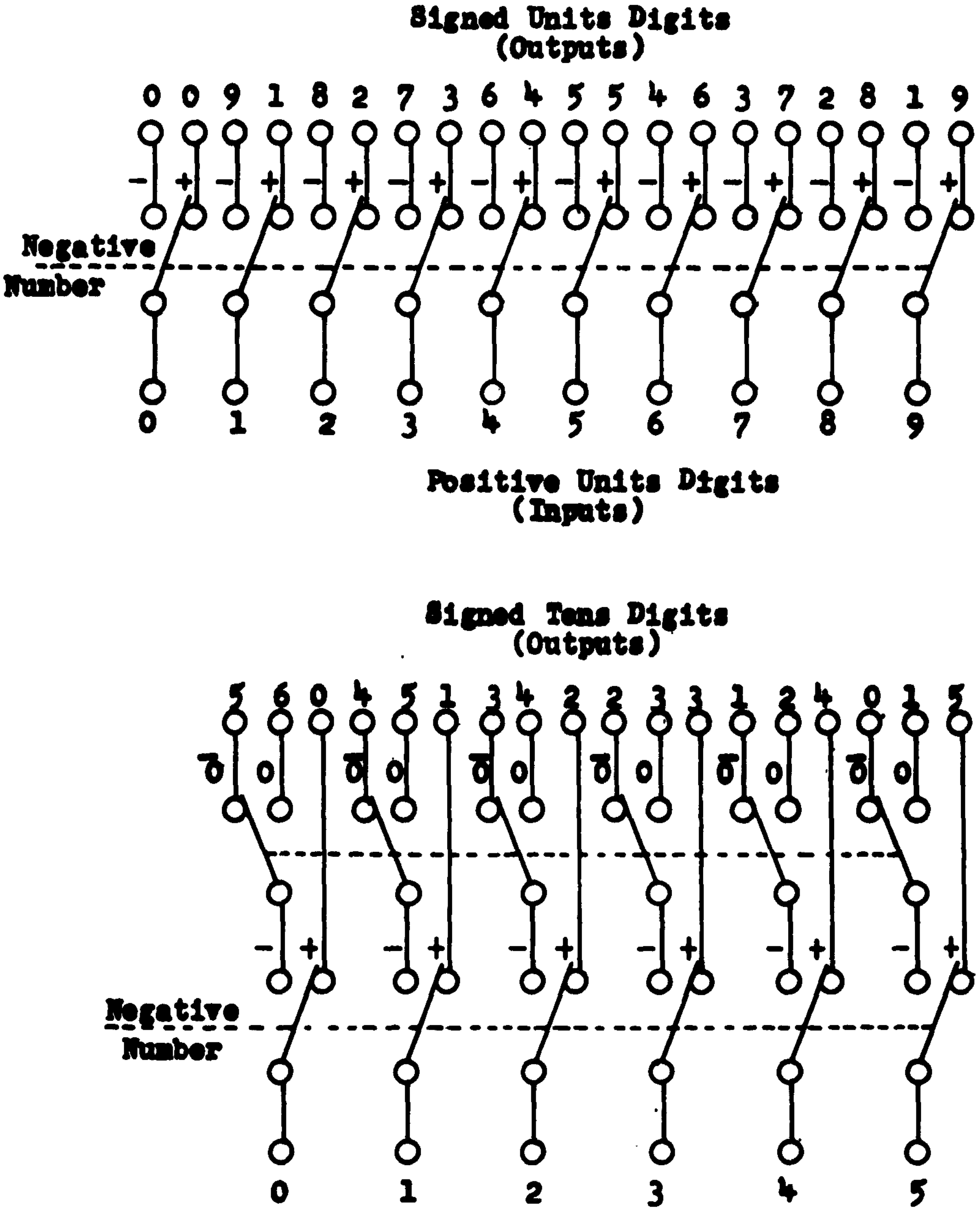


Figure 15

Switching-Circuit Implementation

of Tables 15 and 16

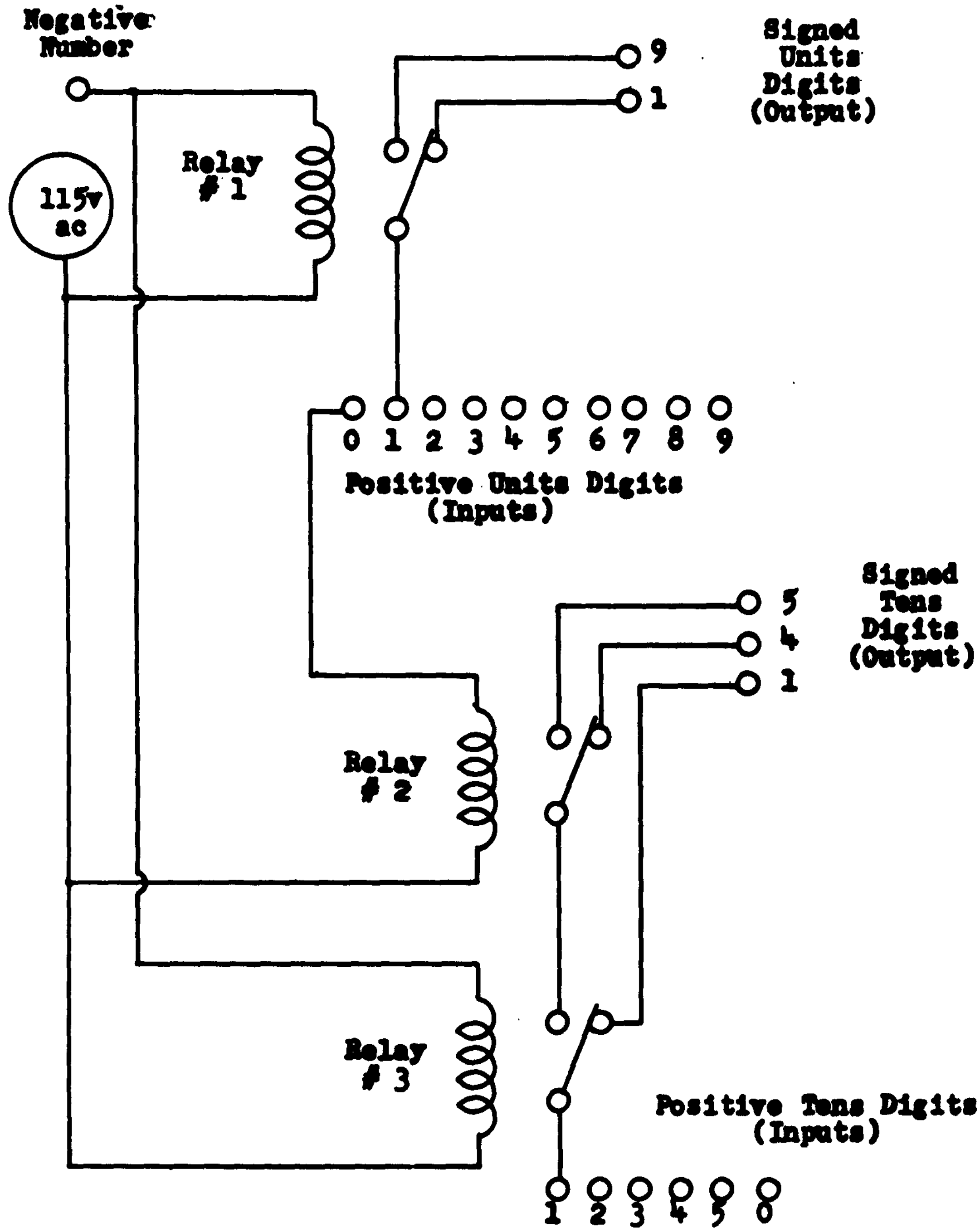


Figure 16

Circuitry to Duplicate the Switching Functions of

Figure 15

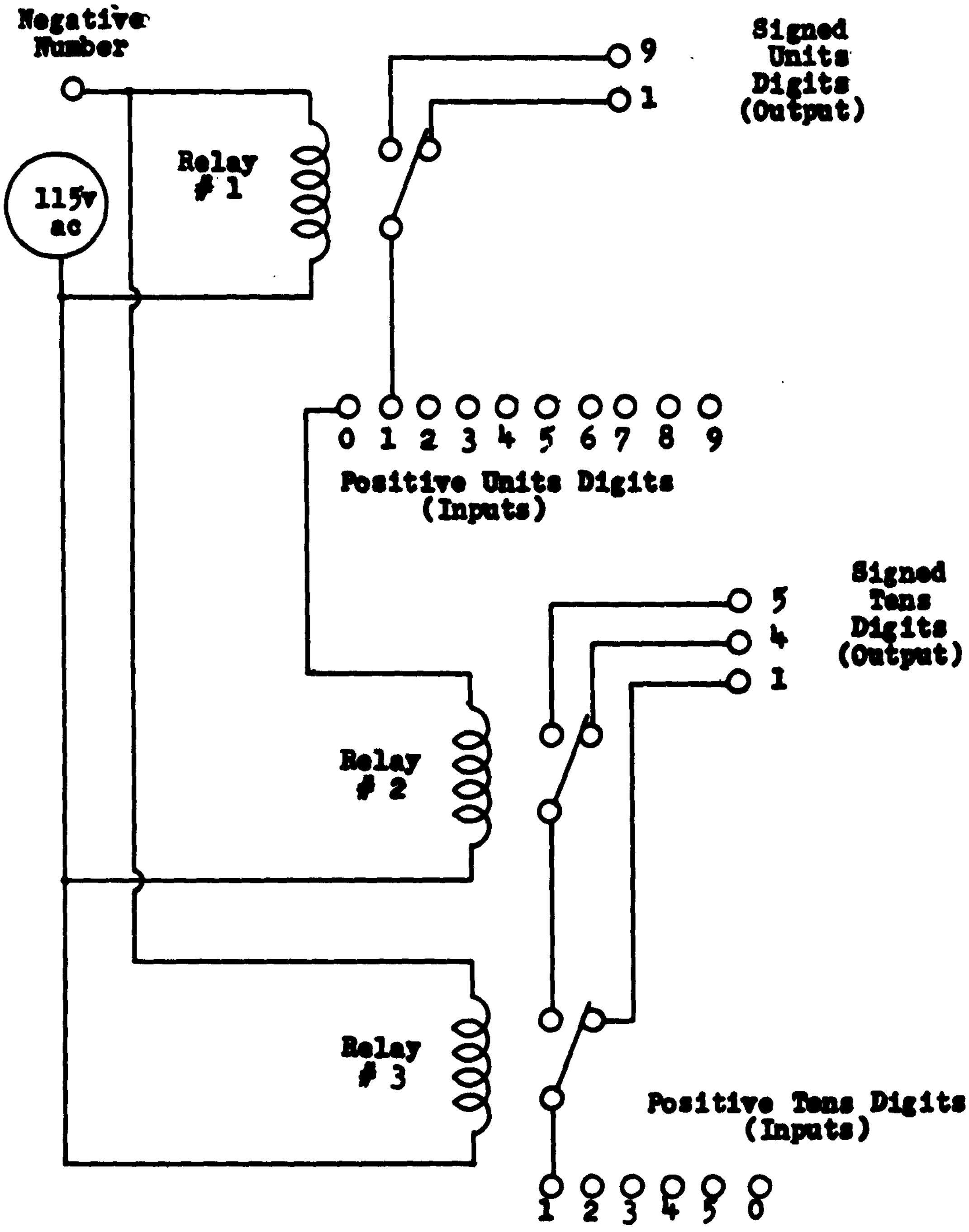


Figure 16

Circuitry to Duplicate the Switching Functions of

Figure 15

as follows:

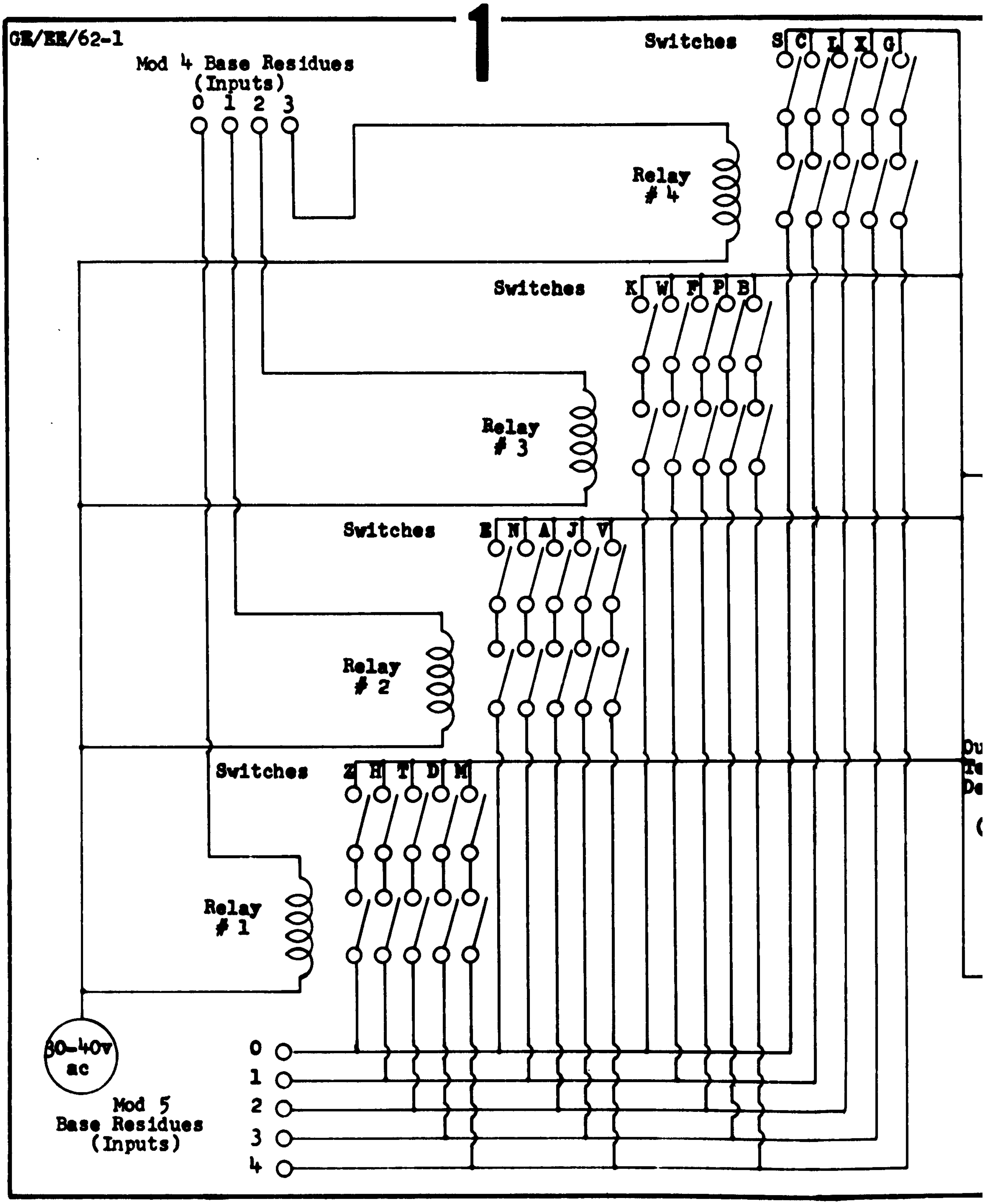
With each of the residue input switches positioned at 2, a ground potential is applied to terminals B, E, and I of terminal strip RRP, and terminals H and C of terminal strip UDP.

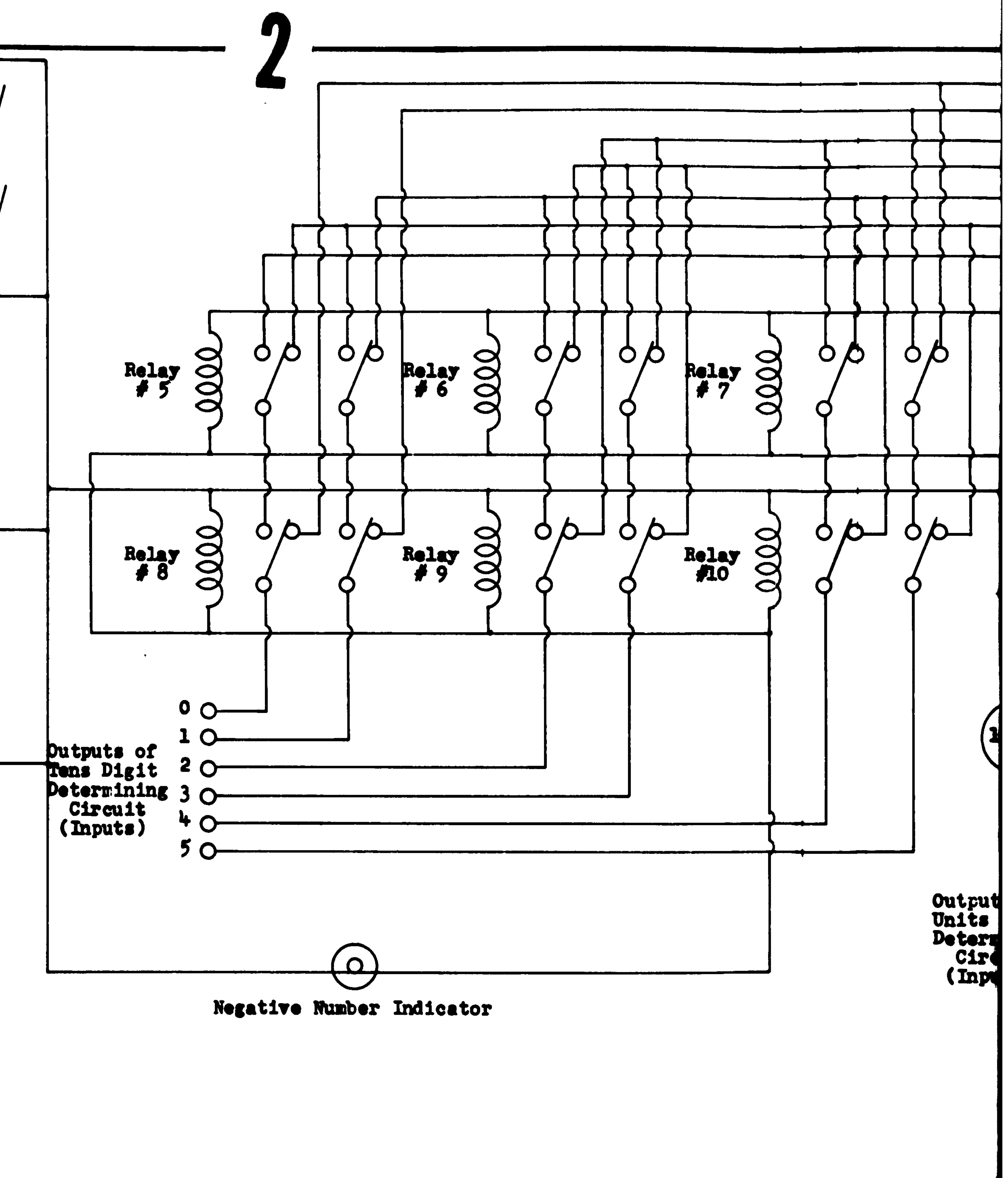
Terminal strip RRP is the input to the Mod 3 Residue Reducing Circuit, so that with power applied through terminal L, and ground applied through terminal B, relays RRR 1, 3, 5, 7, and 9 are energized. Then ground is applied, through terminals E and I, to terminals A and E of terminal strip RRJ, terminals A and I of terminal strip TDP, and terminals A and C of terminal strip RDP.

Terminal strip UDP is the input to the Units Digit Determining Circuit, so that with power applied through terminal J, and ground applied through terminal C, relays UDR 5 and 6 are energized. Then ground is applied, through terminal H, to terminal C of terminal strip UDJ, and terminal C of terminal strip RSP.

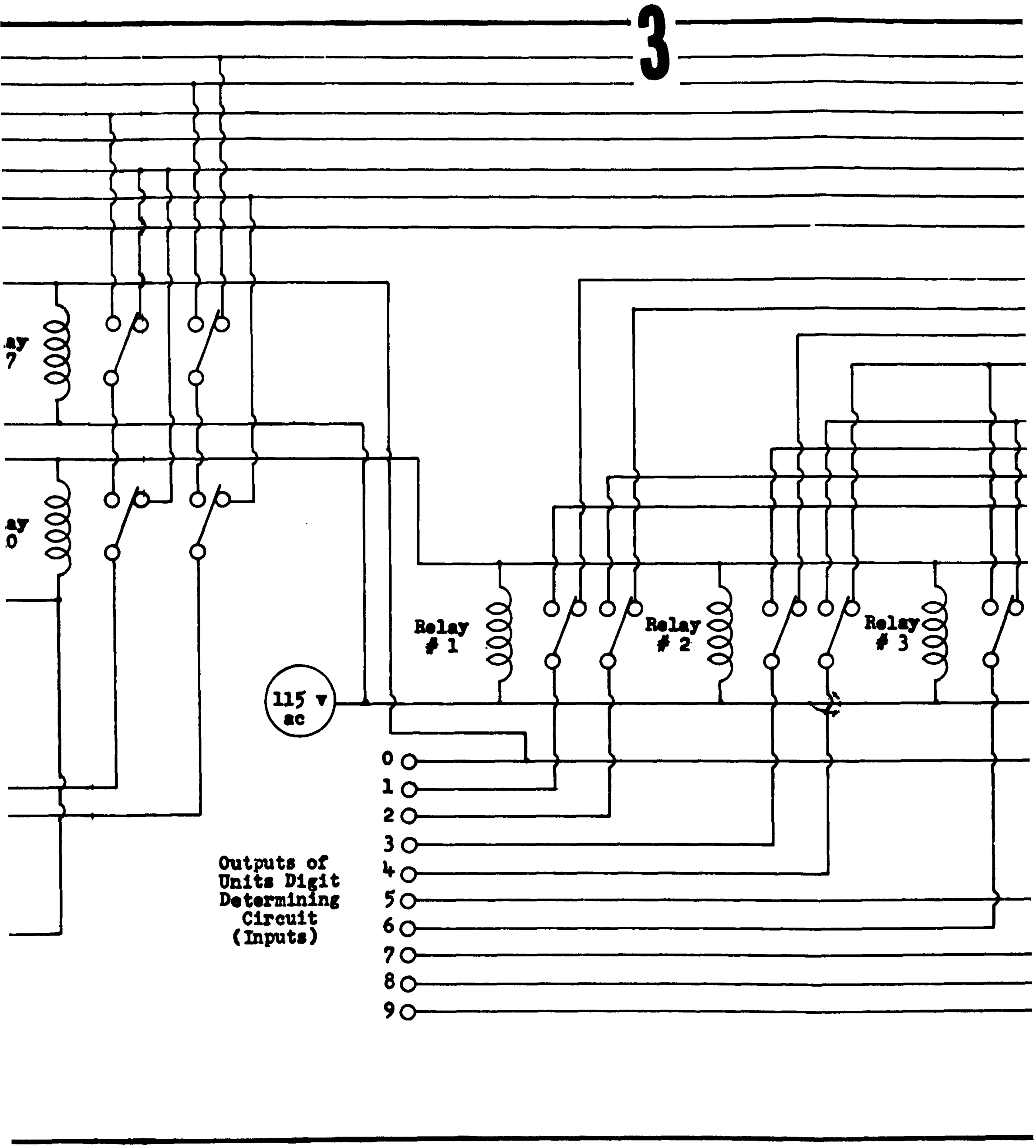
Terminal strip TDP is the input to the Tens Digit Determining Circuit, so that with power applied through terminal N, and ground applied through terminal I, relays TDR 1 and 2 are energized. Then ground is applied, through terminal A, to terminal A of terminal strip UDJ, and terminal J of terminal strip RSP.

Terminal strip RDP is the input to the sign determining portion of the Range and Sign Determining Circuit, so that


GE/EE/62-1
1
Mod 4 Base Residues
(Inputs)
0 1 2 3
Switches S C L X G
Relay
4
Switches K W F P B
Relay
3
Switches E N A J V
Relay
2
Switches Z H T D M
Relay
1
30-40v
ac
Mod 5
Base Residues
(Inputs)
0
1
2
3
4



2
Relay # 5
Relay # 6
Relay # 7
Relay # 8
Relay # 9
Relay #10
0
1
2
3
4
5
Outputs of Tens Digit Determining Circuit (Inputs)
Negative Number Indicator


3
Relay # 1
Relay # 2
Relay # 3
115 v ac
0
1
2
3
4
5
6
7
8
9
Outputs of Units Digit Determining Circuit (Inputs)

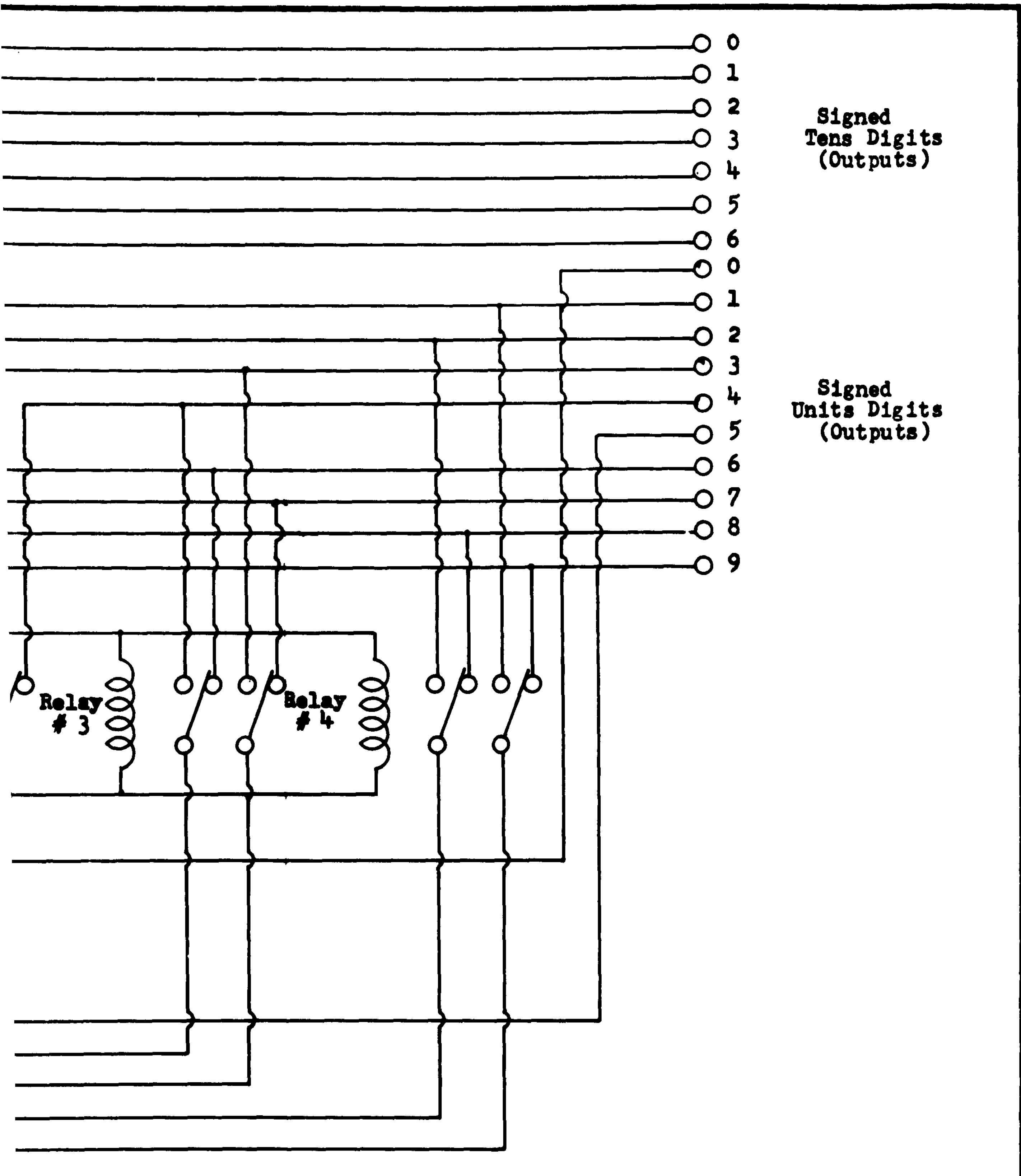


Figure 17

Range and Sign Determining Circuit

with power applied through terminal H, and ground applied through terminal A, relay RDR 1 is energized. Then ground is applied, through terminal C, to terminal A of terminal strip RDJ.

If the number represented is negative, as predetermined by the Sign Determination Switch Bank setting (i.e., at least switch A is closed), ground is applied to terminal Q of terminal strip RSP. If the number is positive (i.e., switches A through Z are open), no potential is applied to terminal Q.

Terminal strip RSP is the input to the range determination portion of the Range and Sign Determining Circuit, so that with power applied through terminal P, and ground applied through terminal Q (i.e., the number represented is negative), relays RSR 1, 2, 3, 4, 8, 9, and 10 are energized. Then ground is applied, through terminals C and J, to terminals G and N of terminal strip RSJ. With power applied to the indicator lights as shown, the number -58 is then indicated.

With no ground applied through terminal Q of terminal strip RSP (i.e., the number represented is positive), all relays of the Range Determination Circuit are deenergized so that ground is applied, through terminals C and J, to terminals J and B of terminal strip RSJ. Then with power applied to the indicator lights as shown, the number 02 is indicated. Referring to Table 14, the decimal number represented by the residue number (2, 2, 2) is -58 in the

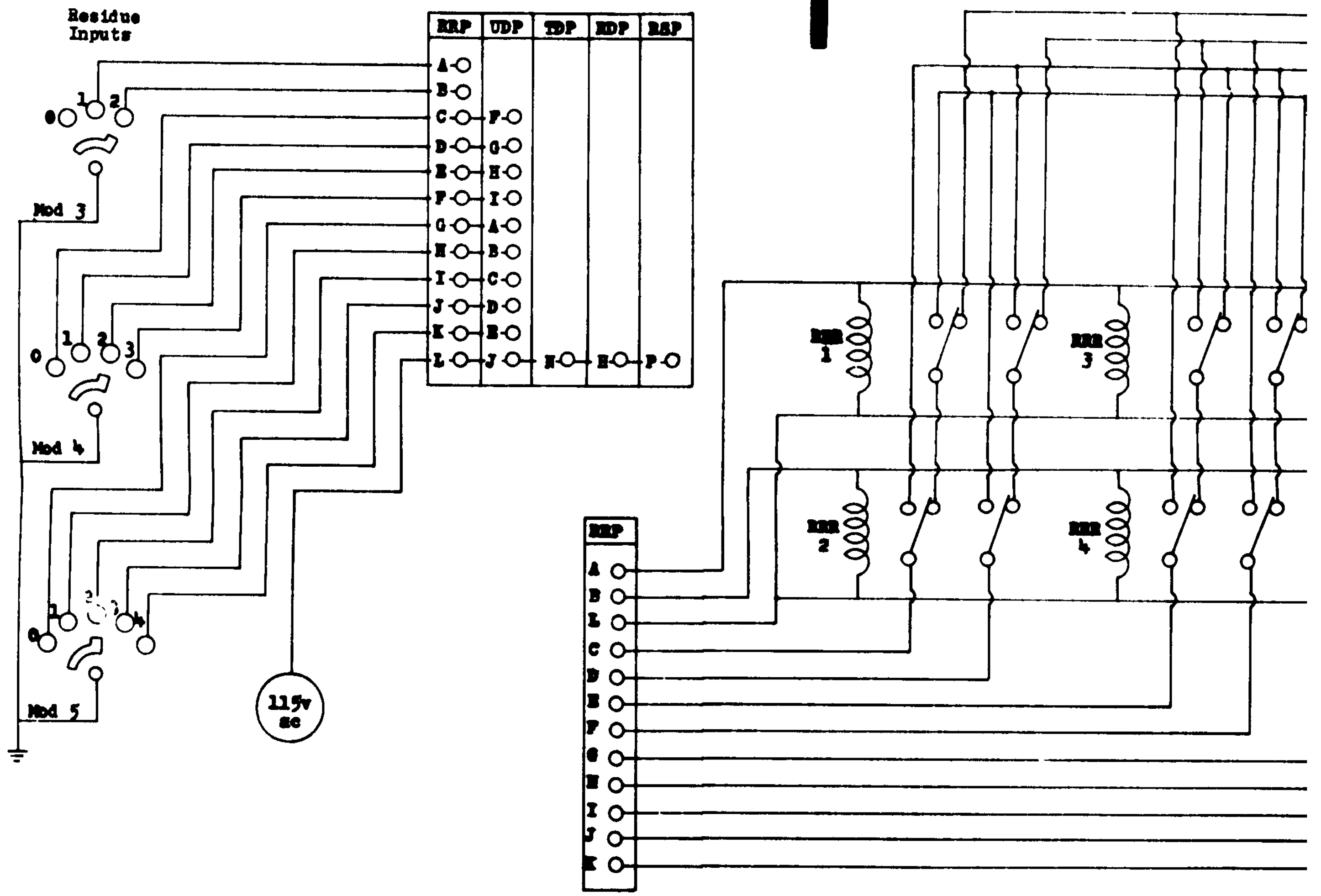
negative range, and 2 in the positive range.

Figure 19 is a picture of the front panel of the Residue-to-Decimal Converter. The range of unique residue number representations is selected by the Range Selection Switch Bank in the lower right portion of the panel. With all switches in the down (off) position, the upper left-hand switch points to the operating range 0 to 59. With the upper left-hand switch (switch A of Figure 17) in the up (on) position, it points to -3, and the next switch points to 56. Thus, the operating range is -3 to 56. Similarly, with the subsequent transition, from down to up, of each switch from left to right, the operating range varies by increments of 3, and the range is indicated by the numbers pointed to by the switches involved in a switch-position inversion. Finally, with all switches in the up (on) position, the operating range is indicated by the lower right-hand switch as -60 to -1.

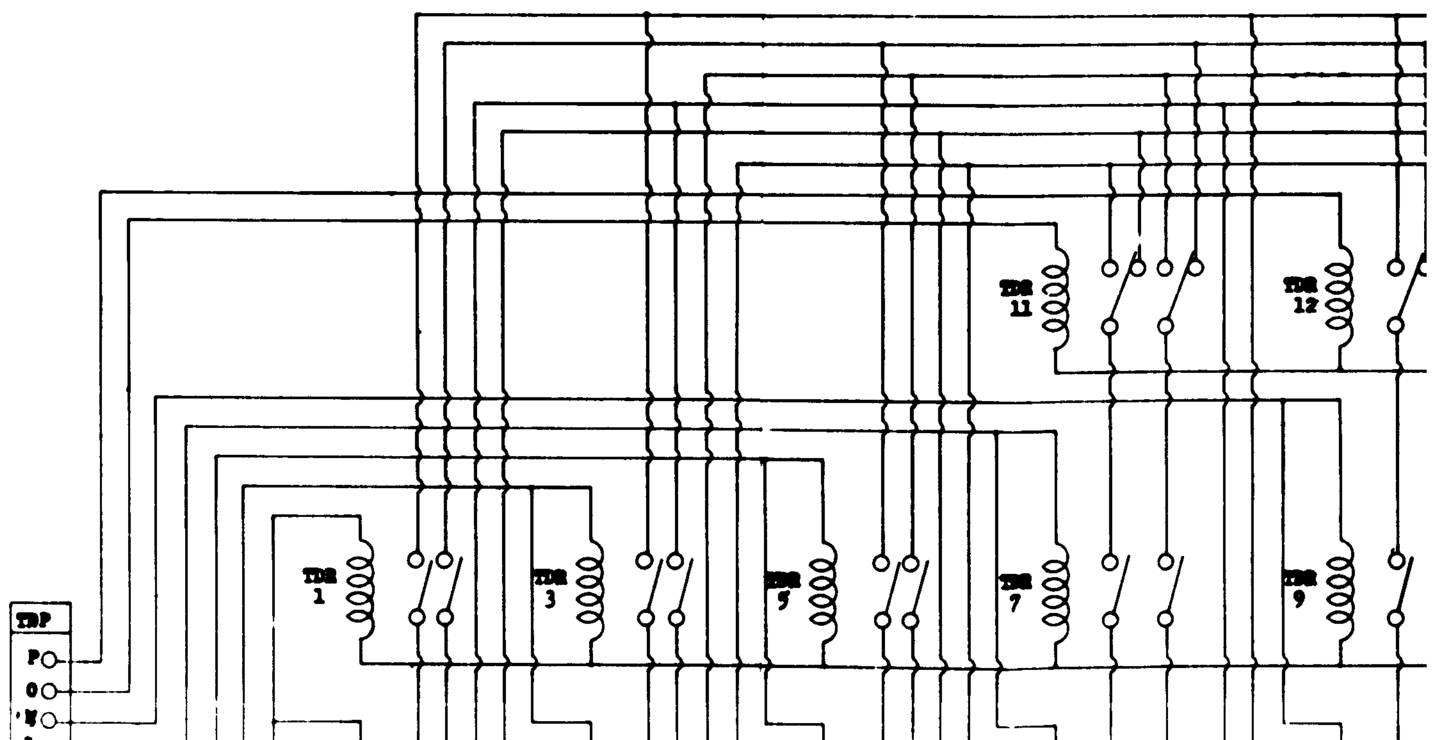
The residue number to be converted is then entered by means of the three rotary Residue Input switches in the upper left portion of the panel. With all inputs entered, the Press-to-Read switch, located to the left of the Range Selection Switch Bank, is pressed. This is a spring-loaded switch designed to prevent the converter circuits from being inadvertently left on for extended periods of time, with consequent overheating of components.

With the Press-to-Read switch energized, the converted decimal number is displayed on the neon indicator bank located

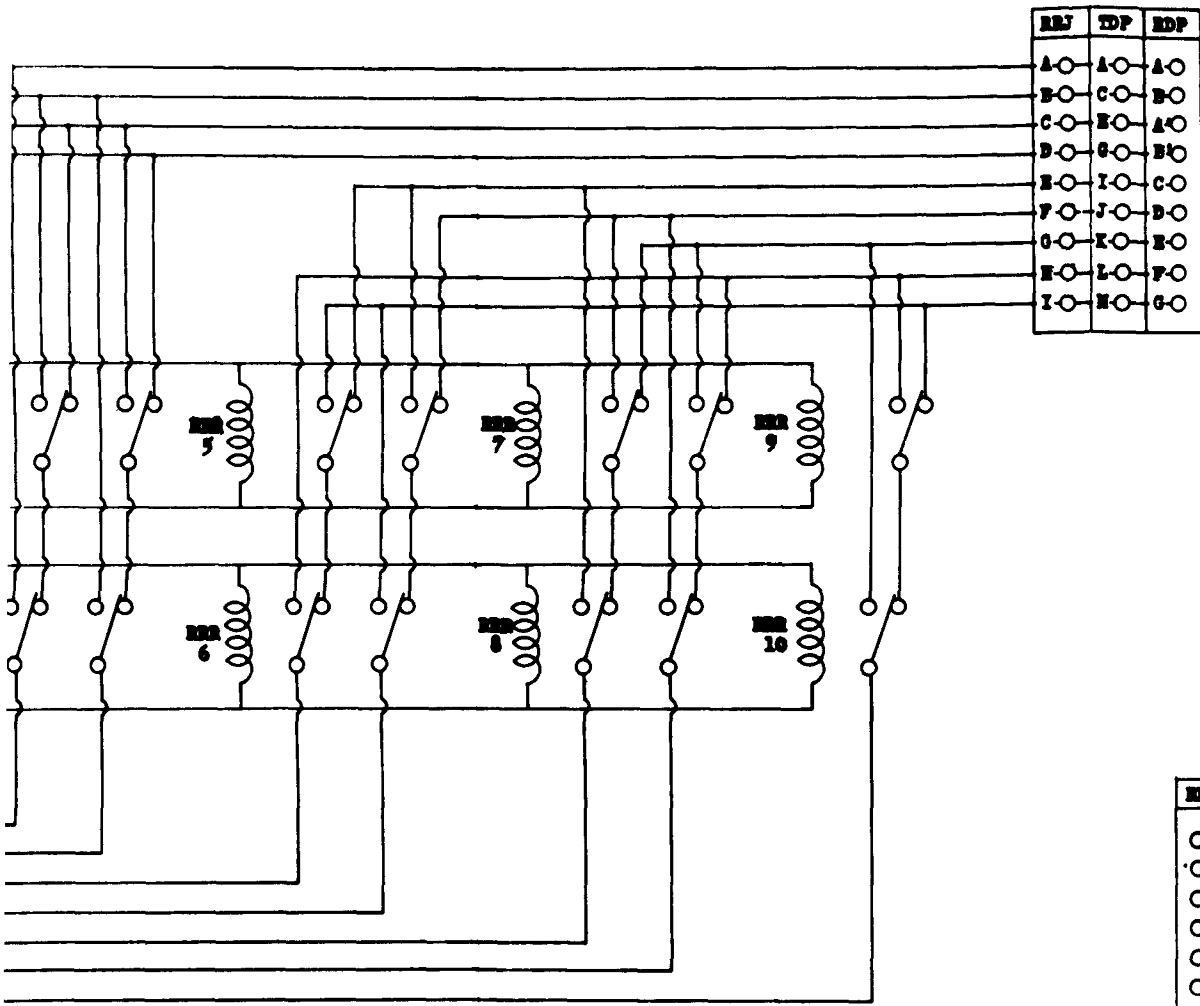
1

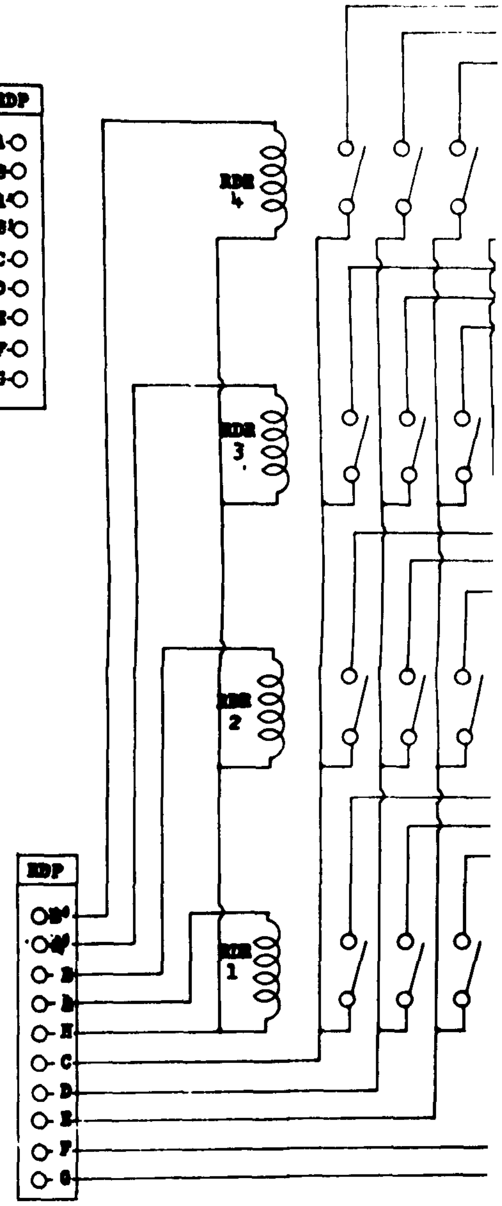


Mod 3 Residue Reducin

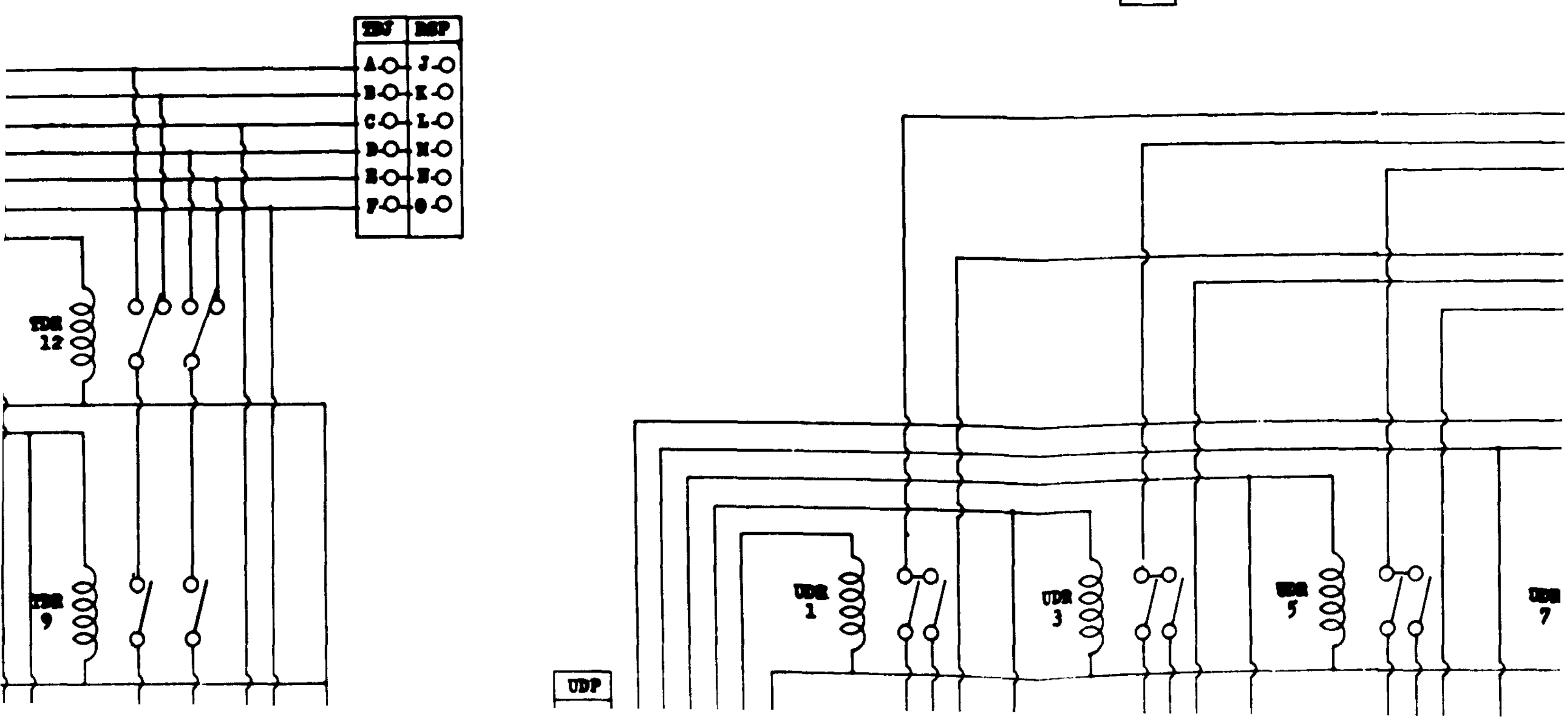


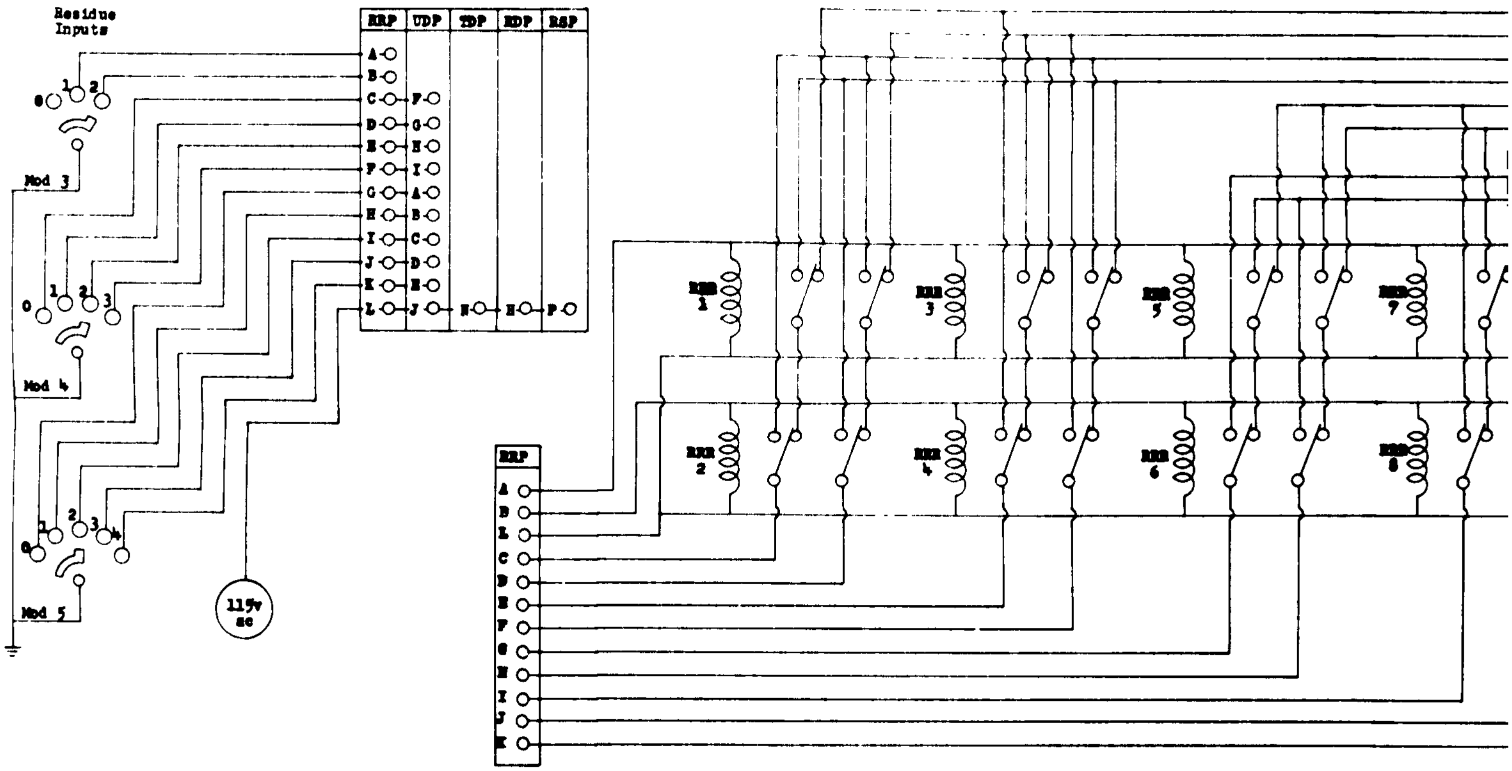
2



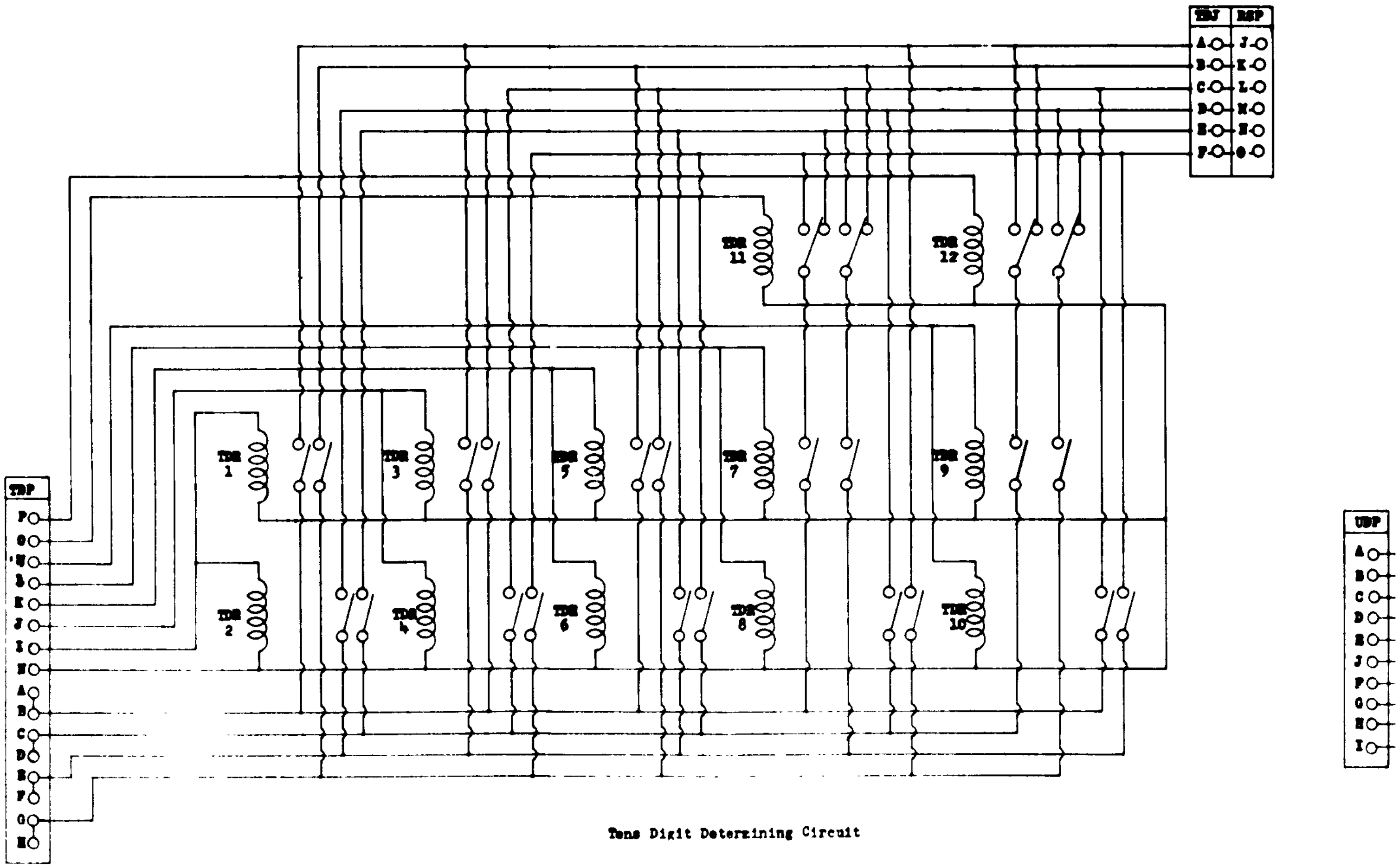


esidue Reducing Circuit



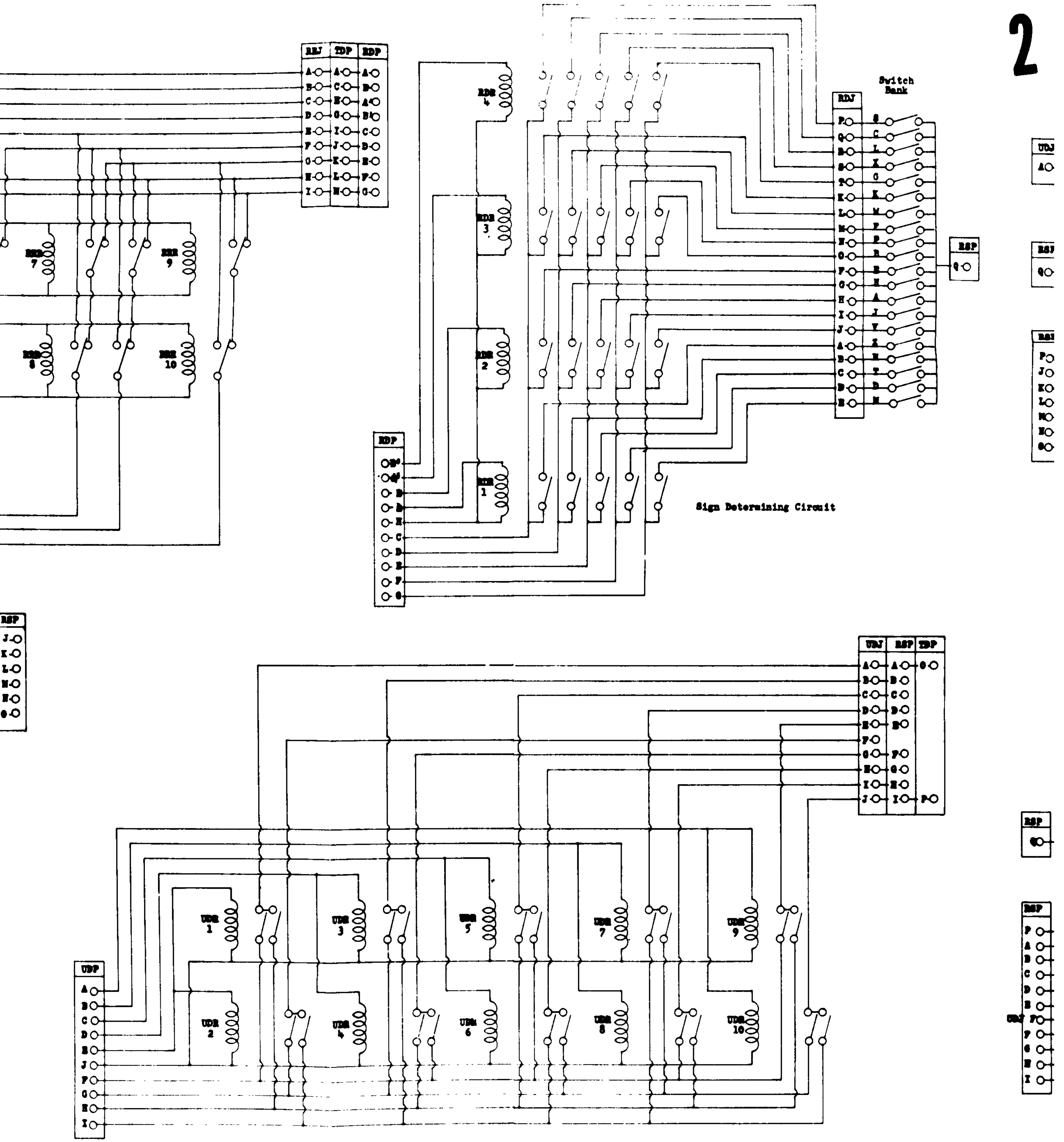


Mod 3 Residue Reducing Circuit



Tens Digit Determining Circuit

2



Sign Determining Circuit

Units Digit Determining Circuit

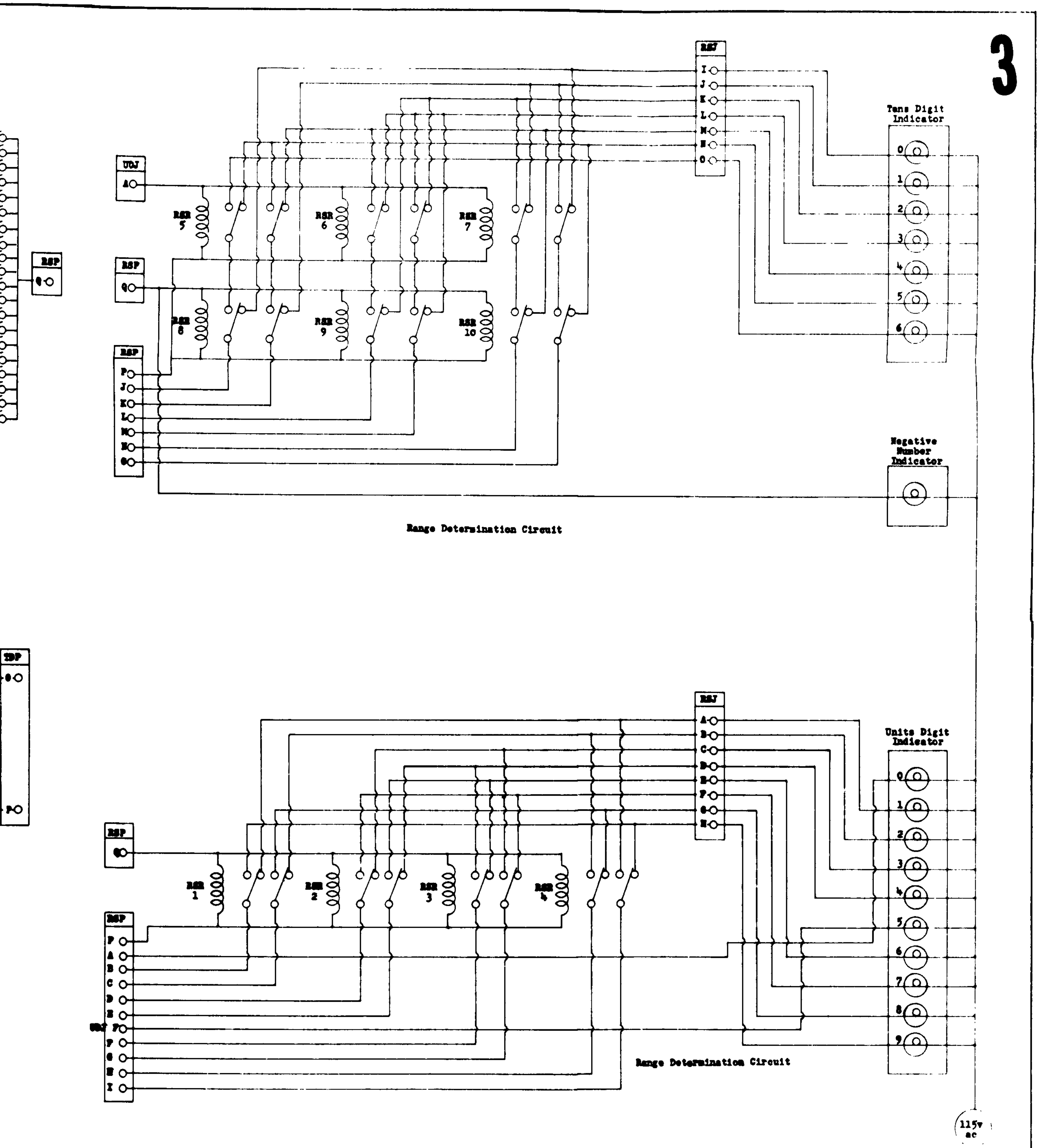


Figure 18

Residue-Decimal Converter

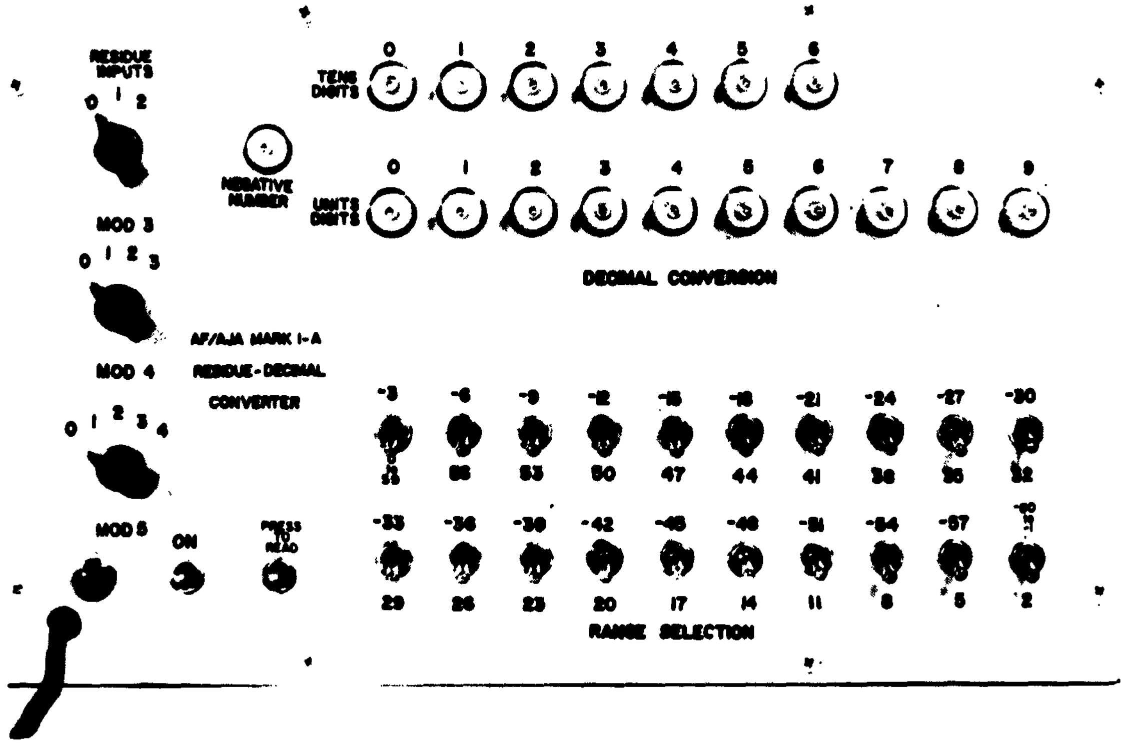


Figure 19

Residue-to-Decimal Converter

Front Panel

in the upper right-hand portion of the panel.

Figures 20 and 21 are right- and left-hand views, respectively, of the interior of the converter.

Figure 20

Converter Interior

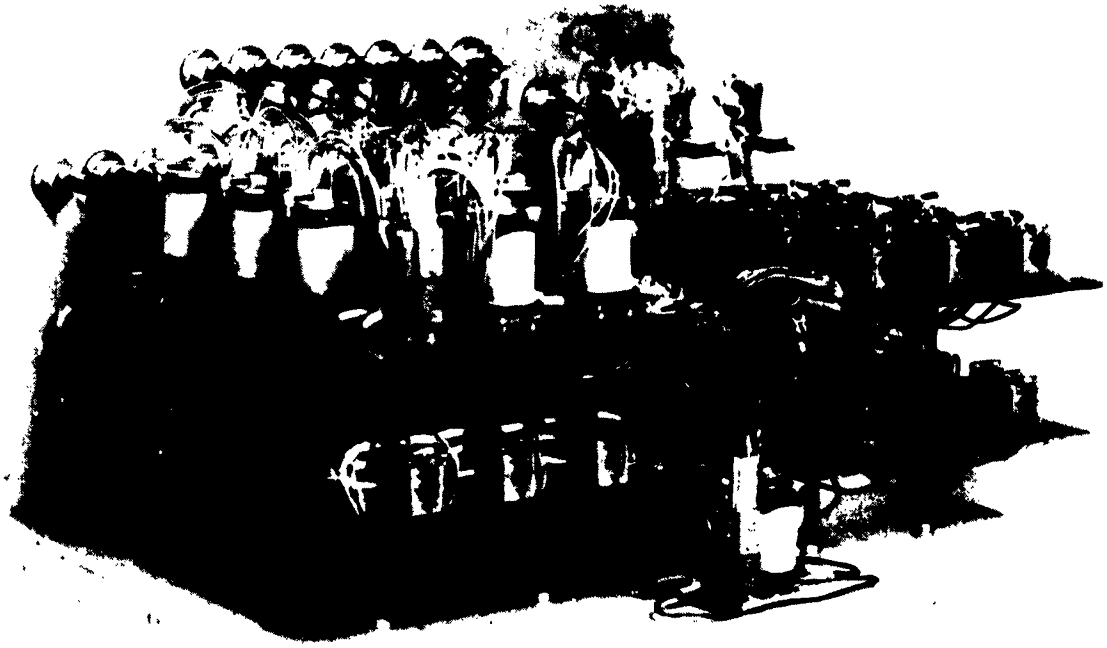


Figure 21

Converter Interior

V. Conclusions

Remaining Problems to be Considered

Aiken and Semon (Ref 1:3-3, 3-4) list seven problems which must be solved before residue number system techniques can be applied to general-purpose digital computers:

1. Capacity
2. Relative magnitude
3. Negative numbers
4. Algebraic sign
5. Fractions
6. Round-off
7. Division

They further indicate that an easily mechanized algorithm for the determination of algebraic sign can be made to yield solutions to the remainder of the first five problems, with the possible exception of round-off, and that solutions to the first six problems will provide a solution to the seventh.

A solution to the problem of relative magnitude is immediately suggested by the algebraic sigh determination algorithm developed in Section IV. That is, the incremental ranges of the limiting modulus within which two residue numbers fall can be compared to determine which represents the set of decimal numbers with the larger relative magnitude. Then, if both numbers fall in the same range, a comparison of the residues of the limiting modulus would determine which

residue number lies farther from the base number, and therefore, represents the decimal number with the larger relative magnitude.

Assuming the suggested solution to the relative magnitude problem to be a practicable one, an extension of the principles involved suggests a solution to the round-off problem. That is, the tens digit of the decimal number represented (in the 3-4-5 residue number system) is determined by the Tens Digit Determining Circuit. Assume that the tens digit of the decimal number represented by a residue number is 2. Then, a comparison of the relative magnitudes of the differences between the given residue number, and the residue number representations of 20 and 30 provides a means to round the given number off to the nearest tens digit. Similarly, comparisons of the relative magnitudes of the differences between the given residue number and the residue number representations of 20, 25, and 30 provide a means of round-off to the nearest 5.

The problem of negative numbers is effectively solved by the algebraic sign determination algorithm developed in Section IV.

A solution to the problem of operating upon fractions in residue number systems by an extension of the algebraic sign determination algorithm is not immediately apparant. All known current methods of operation upon a fraction require first a reduction of the fraction to a decimal (radix) number.

The notion of a radix point, however, is directly associated with positional notation (Ref 1:3-4), and as demonstrated in Section II, residue number systems have zero-order positional significance. As a sidelight, it is noted here that, should some method of solution to the problem of fractions be developed, which either encompasses the difficulty of radix point determination, or translates it to some other parameter which can be absorbed by the device, then that solution would almost certainly be applicable to the first method of solution to the capacity problem, as developed in Section IV, wherein all decimal numbers within the unique range of a residue number system could be treated as fractions of M, the product of the moduli.

Several problems of interest, then, are suggested for possible future investigations in the area of residue number system applications.

1. Development of a practical solution to the problem of relative magnitude.

2. Development of a practical solution to the problem of round-off.

3. Development of a practical solution to the problem of fractions, with an investigation into applicability of any solution developed to the suggested solution to the problem of capacity.

4. Assuming practicable accomplishment of 1 through 3 above, development of a practical solution to the problem

of division.

Conclusions

The summary of residue number systems contained in Section II was included under the assumption that the reader possesses a reasonable familiarity with the concepts discussed. It is a slightly different approach to the subject from that conventionally taken, however, and is intended to function as a springboard into the attack made upon the residue-to-decimal number conversion problem.

In solving the residue-to-decimal conversion problem, it was necessary to attack three problems: The capacity problem; the algebraic sign determination problem; and the machine language to human language translation problem.

The solutions developed to these problems are completely general, and can be applied to any range of positive and negative numbers, limited only by increments of the smallest modulus of the system, in any residue number system. Simplifications utilizing the characteristics of certain classes of moduli have been suggested, but these are not essential to a mechanical realization of the principles developed.

The relay device constructed, and described in conjunction with the development of the principles involved, demonstrates the applicability of the general principles to a specific residue number system.

Bibliography

1. Aiken, H. and W. Semon. _Advanced Digital Computer Logic_. WADC-TR-59-472. Dayton, Ohio: Wright Air Development Center, W-PAFB, 1959.

2. Cowles, W. H. H. and J. E. Thompson. _Algebra for Colleges and Engineering Schools_. New York: D. Van Nostrand Company, Inc., 1948.

3. Culbertson, J. T. _Mathematics and Logic for Digital Devices_. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1959.

4. Ore, O. _Number Theory and its History_. New York: McGraw-Hill Book Company, Inc., 1948.

5. Vinogradov, I. M. _An Introduction to the Theory of Numbers_. (Trans. by Helen Popova) London: Pergamon Press, 1955.

Vita

Arthur James Altenburg was born on 9 July 1931 in Jamaica, New York, the son of Otto James Altenburg, and Mildred Saltzwiedel Altenburg. After completing his work in 1949 at Hempstead High School, Hempstead, New York, he enrolled at Pratt Institute of Technology, Broodklyn, New York. He served in the U. S. Navy as an enlisted man from 1951 to 1953. During that time, he won a Secretary of the Navy appointment to the United States Naval Academy, from which he graduated in June 1957 with the degree of Bachelor of Science. He was commissioned as Lieutenant in the USAF upon graduation. His military assignment prior to assignment to the Institute of Technology was with the 47th Armament and Electronics Squadron.

Permanent Address: 9755 52 Ave. N.
St. Petersburg, 8
Florida

This thesis was typed by Capt. Arthur J. Altenburg